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One loop renormalization of spontaneously broken U(2) gauge theory on noncommutative spacetime

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Abstract: We examine the renormalizability problem of spontaneously broken non-abelian gauge theory on noncommutative spacetime. We show by an explicit analysis of the U(2) case that ultraviolet divergences can be removed at one loop level with the same limited number of renormalization constants as required on commutative spacetime. We thus push forward the efforts towards constructing realistic models of gauge interactions on noncommutative spacetime.

Keywords: Renormalization Regularization and Renormalons, Spontaneous Symmetry Breaking, Gauge Symmetry, Beyond Standard Model.

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Contents

1. Introduction

There have been intense activities in noncommutative (NC) field theory since it was found to arise naturally as a specific limit of string theory [1]. But NC field theory is also interesting in its own right both as a theory which may be relevant to the real world and as a quantum structure on NC spacetime which is very distinct from the one built on the ordinary commutative spacetime. Concerning this, we may mention two issues among others, the unitarity and causality problem [2] and the ultraviolet-infrared mixing [3]. The new interactions and Lorentz violation introduced by noncommutativity also lead to novel phenomenological implications [4], some of which are quite different from those of ordinary new physics beyond the standard model.

Supposing the idea of NC spacetime is physically relevant, it would be desirable to consider whether it is possible to extend the standard model of the electroweak and strong interactions to NC spacetime. It is not completely clear at the moment how to construct such a realistic model due to restrictions on gauge groups from the closure of algebra [5], on possible representations that can be well defined [6] for a product of gauge groups, and potential obstacles in anomaly cancellation that becomes more restrictive [7], and so on [8]. But it is of no doubt that such a model should be perturbatively renormalizable so that gauge symmetry can be maintained order by order in the renormalized theory. Although there is no general proof on renormalizability of gauge theory on NC spacetime for the time being, explicit analyses are indeed available up to some order in some models. It has been shown that exact U(1) [9] and U(N) [10] gauge theories are renormalizable at one loop. The renormalizability of the real ϕ^4 theory has even been confirmed up to two loops [11]. But theories with spontaneous symmetry breaking are more subtle. At first glance one might imagine that the divergence problem cannot be worse compared to the commutative case because of oscillating factors introduced by the star product. But actually renormalizability depends on delicate cancellation of seemingly different sources of divergences which is governed by Ward identities. It is not self-evident at all whether this well-weighted arrangement still persists in NC theories. This is especially true of spontaneously broken theories. As pointed out in ref. [12], there are already problems for spontaneously broken global symmetries; namely, the Goldstone theorem holds valid at one loop level for the NC U(N) linear σ model with a properly ordered potential, but not for the O(N) one (except for $N = 2$ which is equivalent to U(1)). An explicit analysis for the spontaneously broken U(1) gauge theory has been given recently in ref. [13] with the positive result that the divergences can be consistently subtracted at one loop. It is the purpose of this work to pursue further along this line by examining the non-abelian case. The motivation for this should be clear from the above discussion; it is the non-abelian case, especially U(2), that is closer to our goal of building up a model of electroweak interactions on NC spacetime. Our positive result should be encouraging to the efforts in this direction.

The paper is organized as follows. In the next section 2 we present the setup of the model in which we will work. In particular we do gauge-fixing and work out its corresponding ghost terms including counterterms. Section 3 contains the explicit result of divergences in one loop 1PI functions. The renormalization constants are then determined in the MS scheme. We summarize in the last section 4 and state the limitations of this work and prospects for further study. The Feynman rules are listed in appendix A, and collected in appendix B are topologically distinct Feynman diagrams for the 1PI functions computed in section 3.

2. The model

2.1 Brief introduction to NC field theory

Throughout this paper, by n dimensional noncommutative spacetime we mean the one that satisfies the following canonical relation,

$$[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu}\,, \tag{2.1}$$

where $\theta_{\mu\nu}$ is a real, antisymmetric, $n \times n$ constant matrix. Following Weyl we can define a function on NC spacetime by the Fourier transform,

$$\hat{f}(\hat{x}) = \frac{1}{(2\pi)^{n/2}} \int d^4k\; e^{ik_\mu \hat{x}^\mu} \tilde{f}(k)\,, \tag{2.2}$$

where the same $\tilde{f}(k)$ simultaneously defines a function on the usual commutative spacetime,

$$f(x) = \frac{1}{(2\pi)^{n/2}} \int d^4k\; e^{ik_\mu x^\mu} \tilde{f}(k)\,. \tag{2.3}$$

This implements what is known as the Weyl-Moyal correspondence. This relationship is preserved by the product of functions if we replace the usual product of functions on commutative spacetime by the following Moyal-$\star$ product,

$$(f_1 \star f_2)(x) = \left[\exp\left(\frac{i}{2}\theta^{\mu\nu}\partial^x_\mu \partial^y_\nu\right) f_1(x) f_2(y)\right]_{y=x}\,. \tag{2.4}$$

Namely, using eq. (2.1) it is straightforward to show that $\hat{f}_1\hat{f}_2$ and $f_1 \star f_2$ share the same Fourier tranform. In this sense we may study a field theory on NC spacetime by studying its counterpart on commutative spacetime with the usual product of functions replaced by the starred one. A different formalism based on the Seiberg-Witten map is developped in refs. [14]. While there are problems such as the UV-IR mixing in the former approach, it is also not clear how to handle with the expansion of θ at quantum level in the latter [15]. In this paper we will work in the former naive formalism.

For convenience we list below some useful properties of the star product of functions which will be freely used in deriving Feynman rules.

$$\begin{aligned}
(f_1 \star f_2) \star f_3 &= f_1 \star (f_2 \star f_3)\,,\\
(f_1 \star f_2)^\dagger &= f_2^\dagger \star f_1^\dagger\,,\\
f_1 \star f_2 &= f_2 \star f_1|_{\theta \to -\theta}\,,\\
\int d^n x\; f_1 \star f_2 &= \int d^n x\; f_2 \star f_1\,,
\end{aligned} \tag{2.5}$$

where the last one holds for functions which vanish fast enough at infinite spacetime.

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2.2 Classical lagrangian

Instead of considering the general U(N) gauge group, we restrict our explicit analysis to the U(2) case. The reason is twofold. First, the U(2) group is close to the standard model and thus physically well motivated. Second, it is algebraically easier to handle while it has a rich enough structure so that we expect the same conclusion should also be appropriate to the general U(N) case.

We begin with the scalar potential which triggers the spontaneous symmetry breakdown of U(2) to U(1),

$$V = -\mu^2 \Phi^\dagger \star \Phi + \lambda \Phi^\dagger \star \Phi \star \Phi^\dagger \star \Phi\,, \qquad \mu^2 > 0\,, \qquad \lambda > 0\,, \tag{2.6}$$

where Φ is in the fundamental representation of U(2). We assume its vacuum expection value is independent of x; without loss of generality we take

$$\Phi = \phi + \phi_0\,, \qquad \phi = \begin{pmatrix} \pi_+ \\ (\sigma + i\pi_0)/\sqrt{2} \end{pmatrix}, \qquad \phi_0 = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{2.7}$$

with $v = \sqrt{\mu^2/\lambda}$. The negative potential in terms of shifted fields is

$$\begin{aligned} \mathcal{L}_{-V} = -\frac{1}{2} m_\sigma^2 \sigma^2 - \lambda \Big\{ & v\sigma(\sigma^2 + \pi_0^2 + 2\pi_-\pi_+) + \pi_-\pi_+\pi_-\pi_+ + \frac{1}{4}(\sigma^4 + \pi_0^4) + \\ & + \pi_-\pi_+(\sigma^2 + \pi_0^2) + \sigma^2\pi_0^2 - \frac{1}{2}\sigma\pi_0\sigma\pi_0 + \pi_-\pi_+ i[\sigma, \pi_0] \Big\}, \end{aligned} \tag{2.8}$$

where $m_\sigma = v\sqrt{2\lambda}$ is the mass of the physical Higgs boson. We have freely ignored terms which vanish upon spacetime integration using properties of the star product. We also suppress the explicit $\star$ notation from now on.

It is convenient to formulate the gauge part by matrix. Denoting

$$t^A = \begin{cases} \frac{1}{2}\sigma^A & \text{for } A = 1,2,3 \\ \frac{1}{2}1_2 & \text{for } A = 0 \end{cases}, \ \text{with} \ \ \mathrm{tr}\,(t^A t^B) = \frac{1}{2}\delta^{AB}\,, \tag{2.9}$$

the gauge field is

$$G_\mu = G_\mu^A t^A = \frac{1}{\sqrt{2}} \begin{pmatrix} A_\mu & W_\mu^+ \\ W_\mu^- & Z_\mu \end{pmatrix}. \tag{2.10}$$

The Yang-Mills lagrangian is

$$\mathcal{L}_G = -\frac{1}{2} \mathrm{Tr}\ G_{\mu\nu} G^{\mu\nu}\,, \qquad G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu - ig[G_\mu, G_\nu]\,, \tag{2.11}$$

where g is the coupling. In terms of physical fields A, Z and $W^\pm$ as to be clear later on, we have

$$\mathcal{L}_G = \mathcal{L}_{2G} + \mathcal{L}_{3G} + \mathcal{L}_{4G}\,, \tag{2.12}$$

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where

$$
\begin{aligned}
\mathcal{L}_{2G} &= -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)^2 - \\
&\quad -\frac{1}{2}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)(\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}), \qquad (2.13)\\
\mathcal{L}_{3G} &= +\frac{ig}{\sqrt{2}}\left(\partial_\mu A_\nu [A^\mu, A^\nu] + \partial_\mu Z_\nu [Z^\mu, Z^\nu]\right) + \\
&\quad +\frac{ig}{\sqrt{2}} A^\mu \left(W^+_\nu \partial_\mu W^{-\nu} + \partial^\nu W^+_\mu W^-_\nu - \partial_\mu W^+_\nu W^{-\nu} - W^+_\nu \partial^\nu W^-_\mu\right) + \\
&\quad +\frac{ig}{\sqrt{2}} Z^\mu \left(W^-_\nu \partial_\mu W^{+\nu} + \partial^\nu W^-_\mu W^+_\nu - \partial_\mu W^-_\nu W^{+\nu} - W^-_\nu \partial^\nu W^+_\mu\right) + \\
&\quad +\frac{ig}{\sqrt{2}} \left(\partial^\mu A^\nu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) + \partial^\mu Z^\nu (W^-_\mu W^+_\nu - W^-_\nu W^+_\mu)\right), \qquad (2.14)\\
\mathcal{L}_{4G} &= +\frac{g^2}{8}\left([A_\mu, A_\nu]^2 + [Z_\mu, Z_\nu]^2\right) + \\
&\quad +\frac{g^2}{4}\left(2W^+_\mu W^-_\nu W^{+\mu} W^{-\nu} - W^-_\nu W^{+\nu} W^-_\mu W^{+\mu} - W^+_\nu W^{-\nu} W^+_\mu W^{-\mu}\right) + \\
&\quad +\frac{g^2}{2}\left([A^\mu, A^\nu] W^+_\mu W^-_\nu + A^\mu A^\nu W^+_\mu W^-_\nu - A^\mu A_\mu W^{+\nu} W^-_\nu\right) + \\
&\quad +\frac{g^2}{2}\left([Z^\mu, Z^\nu] W^-_\mu W^+_\nu + Z^\mu Z^\nu W^-_\mu W^+_\nu - Z^\mu Z_\mu W^{-\nu} W^+_\nu\right) + \\
&\quad +\frac{g^2}{2} A^\mu \left(2W^+_\nu Z_\mu W^{-\nu} - W^+_\nu Z^\nu W^-_\mu - W^+_\mu Z^\nu W^-_\nu\right). \qquad (2.15)
\end{aligned}
$$

The covariant kinetic lagrangian for the scalar is

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger D^\mu \Phi\,, \qquad D_\mu \Phi = \partial_\mu \Phi - i g G_\mu \Phi\,. \qquad (2.16)$$

In terms of physical fields it can be cast in the form,

$$\mathcal{L}_\Phi = \mathcal{L}_{2\phi} + \mathcal{L}_{G\ \mathrm{mass}} + \mathcal{L}_{\phi G} + \mathcal{L}_{G\phi\phi} + \mathcal{L}_{GG\phi} + \mathcal{L}_{GG\phi\phi}\,, \qquad (2.17)$$

where

$$
\begin{aligned}
\mathcal{L}_{2\phi} &= \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \pi_0)^2 + \partial_\mu \pi_+ \partial^\mu \pi_-\,, \qquad (2.18)\\
\mathcal{L}_{\mathrm{G\ mass}} &= \frac{1}{2} m_Z^2 Z_\mu Z^\mu + m_W^2 W^+_\mu W^{-\mu}\,, \qquad (2.19)\\
\mathcal{L}_{\phi G} &= -m_Z Z^\mu \partial_\mu \pi_0 + i m_W (W^-_\mu \partial^\mu \pi_+ - W^+_\mu \partial^\mu \pi_-), \qquad (2.20)\\
\mathcal{L}_{G\phi\phi} &= +\frac{ig}{\sqrt{2}} A^\mu (\partial_\mu \pi_+ \pi_- - \pi_+ \partial_\mu \pi_-) + \\
&\quad +\frac{ig}{2\sqrt{2}} Z^\mu \left((\partial_\mu \sigma \sigma - \sigma \partial_\mu \sigma) + (\partial_\mu \pi_0 \pi_0 - \pi_0 \partial_\mu \pi_0)\right) + \\
&\quad +\frac{g}{2\sqrt{2}} Z^\mu \left(\pi_0 \partial_\mu \sigma - \partial_\mu \pi_0 \sigma + \partial_\mu \sigma \pi_0 - \sigma \partial_\mu \pi_0\right) +
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{ig}{2}W_\mu^+(\partial^\mu\sigma\pi_- - \sigma\partial^\mu\pi_-) + \frac{ig}{2}W_\mu^-(\partial^\mu\pi_+\sigma - \pi_+\partial^\mu\sigma) + \\
&+\frac{g}{2}W_\mu^+(\pi_0\partial^\mu\pi_- - \partial^\mu\pi_0\pi_-) + \frac{g}{2}W_\mu^-(\partial^\mu\pi_+\pi_0 - \pi_+\partial^\mu\pi_0)\,, \qquad (2.21)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{GG\phi} &= +\frac{g^2v}{2\sqrt{2}}\left(A^\mu(\pi_+W_\mu^- + W_\mu^+\pi_-) + Z^\mu(W_\mu^-\pi_+ + \pi_-W_\mu^+)\right) + \\
&\quad +\frac{g^2v}{2}(W_\mu^-W^{+\mu} + Z_\mu Z^\mu)\sigma\,, \qquad (2.22)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{GG\phi\phi} &= +\frac{g^2}{2}\pi_+\pi_-\left(A_\mu A^\mu + W_\mu^+W^{-\mu}\right) + \\
&\quad +\frac{g^2}{4}\left(\sigma^2 + \pi_0^2 + i[\pi_0,\sigma]\right)\left(Z_\mu Z^\mu + W_\mu^-W^{+\mu}\right) + \\
&\quad +\frac{g^2}{2\sqrt{2}}\sigma\left((\pi_-A^\mu W_\mu^+ + W_\mu^-A^\mu\pi_+) + (\pi_-W_\mu^+Z^\mu + Z^\mu W_\mu^-\pi_+)\right) + \\
&\quad +\frac{ig^2}{2\sqrt{2}}\pi_0\left((\pi_-A^\mu W_\mu^+ - W_\mu^-A^\mu\pi_+) + (\pi_-W_\mu^+Z^\mu - Z^\mu W_\mu^-\pi_+)\right), \qquad (2.23)
\end{aligned}
$$

and $m_W = gv/2$ and $m_Z = gv/\sqrt{2}$ are $W^\pm$ and Z masses respectively.

The action defined by the classical lagrangian

$$
\mathcal{L}_{\rm class} = \mathcal{L}_G + \mathcal{L}_\Phi + \mathcal{L}_{-V} \qquad (2.24)
$$

is invariant under the generalized, starred U(2) transformation,

$$
\begin{aligned}
G_\mu &\to G'_\mu = U\star G_\mu\star U^{-1} + ig^{-1}U\star\partial_\mu U^{-1}\,, \\
\Phi &\to \Phi' = U\star\Phi\,, \qquad (2.25)
\end{aligned}
$$

where $U = \exp(ig\eta(x))_\star$ and we have restored the explicit star notation for clearness.

2.3 Gauge fixing and ghost terms

To make the theory well-defined and to quantize it, we should do gauge fixing and include its corresponding ghost terms. Since the quadratic terms in the action remain the same on NC spacetime, it is easy to expect how to generalize the gauge fixing procedure; namely we replace the usual product by the starred one (again suppressing the notation from now on),

$$
\begin{aligned}
\mathcal{L}_{\rm g.f.} &= -\frac{1}{\xi}{\rm Tr}(ff)\,, \\
f &= \partial^\mu G_\mu + ig\xi(\phi^\dagger t^A\phi_0 - \phi_0^\dagger t^A\phi)t^A\,. \qquad (2.26)
\end{aligned}
$$

To construct the ghost terms we first generalize the BRS transformation to NC spacetime,

$$
\begin{aligned}
\delta G_\mu &= \epsilon(\partial_\mu c + ig[c, G_\mu])\,, \\
\delta\phi &= \epsilon igc\Phi\,, \qquad \delta\phi^\dagger = -\epsilon ig\Phi^\dagger c\,, \\
\delta c &= \epsilon igcc\,, \qquad (2.27)
\end{aligned}
$$

where ϵ is an infinitesimal Grassmann constant and c is the ghost field. We have

$$\mathcal{L}_{\text{ghost}} = -2\text{Tr}(\bar{c}\mathsf{s}f)\,, \tag{2.28}$$

where $\epsilon\mathsf{s}f$ is the BRS transformation of the gauge fixing function f and thus,

$$\mathsf{s}f = \partial^\mu(\partial_\mu c + ig[c, G_\mu]) + g^2\xi(\Phi^\dagger c t^A \phi_0 + \phi_0^\dagger t^A c \Phi)t^A\,. \tag{2.29}$$

Noting that $\mathsf{s}^2 f = 0$, the BRS invariance of the sum $\mathcal{L}_{\text{g.f.}} + \mathcal{L}_{\text{ghost}}$ is then guaranteed by requiring

$$\delta\bar{c} = -\frac{1}{\xi}\epsilon f\,. \tag{2.30}$$

We parametrize the ghost fields as follows,

$$c = \frac{1}{\sqrt{2}}\begin{pmatrix} c_A & c_+ \\ c_- & c_Z \end{pmatrix}\,, \qquad \bar{c} = \frac{1}{\sqrt{2}}\begin{pmatrix} \bar{c}_A & \bar{c}_+ \\ \bar{c}_- & \bar{c}_Z \end{pmatrix}. \tag{2.31}$$

Then, the explicit forms of $\mathcal{L}_{\text{g.f.}}$ and $\mathcal{L}_{\text{ghost}}$ are,

$$\begin{aligned}
\mathcal{L}_{\text{g.f.}} &= -\frac{1}{2\xi}\left((\partial^\mu A_\mu)^2 + (\partial^\mu Z_\mu)^2\right) - \frac{1}{\xi}\partial^\mu W_\mu^+ \partial^\nu W_\nu^- - \\
&\quad -\frac{1}{2}\xi m_Z^2 \pi_0^2 - \xi m_W^2 \pi_+\pi_- - \\
&\quad -m_Z\pi_0\partial^\mu Z_\mu - i m_W(\pi_-\partial^\mu W_\mu^+ - \pi_+\partial^\mu W_\mu^-)\,, && (2.32)\\
\mathcal{L}_{\text{ghost}} &= \mathcal{L}_{c\bar{c}} + \mathcal{L}_{\phi c\bar{c}} + \mathcal{L}_{Gc\bar{c}}\,, && (2.33)\\
\mathcal{L}_{c\bar{c}} &= -\bar{c}_A\partial^2 c_A - \bar{c}_Z(\partial^2 + \xi m_Z^2)c_Z - \bar{c}_-(\partial^2 + \xi m_W^2)c_+ - \bar{c}_+(\partial^2 + \xi m_W^2)c_-\,, && (2.34)\\
\mathcal{L}_{\phi c\bar{c}} &= -\frac{1}{4}\xi g^2 v\bar{c}_-\left(\sqrt{2}c_A\pi_+ + c_+(\sigma + i\pi_0)\right) - \\
&\quad -\frac{1}{4}\xi g^2 v\bar{c}_+\left(\sqrt{2}\pi_- c_A + (\sigma - i\pi_0)c_-\right) - \\
&\quad -\frac{1}{4}\xi g^2 v\bar{c}_Z\left(\sqrt{2}(\pi_- c_+ + c_-\pi_+) + \{\sigma, c_Z\} + i[c_Z, \pi_0]\right)\,, && (2.35)\\
\mathcal{L}_{Gc\bar{c}} &= -\frac{ig}{\sqrt{2}}\bar{c}_A\partial^\mu\left([c_A, A_\mu] + (c_+W_\mu^- - W_\mu^+c_-)\right) - \\
&\quad -\frac{ig}{\sqrt{2}}\bar{c}_Z\partial^\mu\left([c_Z, Z_\mu] + (c_-W_\mu^+ - W_\mu^-c_+)\right) - \\
&\quad -\frac{ig}{\sqrt{2}}\bar{c}_-\partial^\mu\left(c_AW_\mu^+ - A_\mu c_+ + c_+Z_\mu - W_\mu^+c_Z\right) - \\
&\quad -\frac{ig}{\sqrt{2}}\bar{c}_+\partial^\mu\left(c_-A_\mu - W_\mu^-c_A + c_ZW_\mu^- - Z_\mu c_-\right)\,. && (2.36)
\end{aligned}$$

Note that the $G\phi$ mixing terms in $\mathcal{L}_{\text{g.f.}}$ are cancelled by $\mathcal{L}_{\phi G}$. The complete Feynman rules are collected in appendix A.

2.4 Renormalization constants and counterterms

To go beyond the lowest order we introduce the following renormalization constants for bare quantities,

$$\begin{aligned} (G_\mu)_{\rm B} &= Z_G^{1/2} G_\mu\,, & (\phi)_{\rm B} &= Z_\phi^{1/2}\phi\,,\\ (g)_{\rm B} &= Z_G^{-1/2} Z_g g\,, & (\lambda)_{\rm B} &= Z_\phi^{-2} Z_\lambda \lambda\,,\\ (\mu^2)_{\rm B} &= Z_\phi^{-1}\mu^2\left(1+\frac{\delta\mu^2}{\mu^2}\right), & (v)_{\rm B} &= Z_\phi^{1/2} v\left(1+\frac{\delta v}{v}\right). \end{aligned} \tag{2.37}$$

Note that the redundant renormalization constant δv will be determined by the additional requirement that the σ tadpole be cancelled at one loop. We have chosen the same renormalization constant for all members of a multiplet. As to be shown below this will be sufficent for removing divergences.

The counterterms introduced by the above substitutions are standard for the linear and quadratic terms since no difference arises as compared to the commutative case. These are listed explicitly in appendeix A together with the rules to obtain counterterms for vertices in $\mathcal{L}_{\rm class}$. We focus below on deriving counterterms in the gauge fixing sector.

Since the gauge fixing function can be chosen at will, we choose it to be given in terms of renormalized quantities,

$$(\xi)_{\rm B} = \xi\,, \qquad (f)_{\rm B} = f = \partial^\mu G_\mu + \frac{igv\xi}{\sqrt{2}}\left(\phi^\dagger t^A \hat{\phi}_0 - \hat{\phi}_0^\dagger t^A \phi\right) t^A\,, \tag{2.38}$$

where $\hat{\phi}_0^\dagger = (0 \quad 1)$. We require that $(D_\mu\Phi)_{\rm B} = Z_\phi^{1/2}(\partial_\mu - igZ_g G_\mu)\Phi$ be covariant under the infinitesimal gauge transformation for renormalized fields,

$$\begin{aligned} \delta G_\mu &= y\partial_\mu \eta + zig[\eta, G_\mu]\,,\\ \delta\Phi &= xig\eta\Phi\,, \qquad \delta\Phi^\dagger = -xig\Phi^\dagger\eta\,. \end{aligned} \tag{2.39}$$

This determines the constants $x = z = Z_g y$. Now consider the BRS transformation for renormalized fields,

$$\begin{aligned} \delta G_\mu &= \epsilon(y\partial_\mu c + zig[c, G_\mu])\,,\\ \delta\phi &= \epsilon xigc\Phi\,, \qquad \delta\phi^\dagger = -\epsilon xig\Phi^\dagger c\,,\\ \delta c &= \epsilon uigcc\,. \end{aligned} \tag{2.40}$$

We find,

$$\mathsf{s}f = \partial^\mu(y\partial_\mu c + zig[c, G_\mu]) + xg^2 v\xi/\sqrt{2}(\Phi^\dagger c t^A \hat{\phi}_0 + \hat{\phi}_0^\dagger t^A c\Phi)t^A\,. \tag{2.41}$$

Note that Φ now contains v in the form of $v + \delta v$. We hope to keep eq. (2.30) intact so that $(\bar{c})_{\rm B} = \bar{c}$. The invariance of the sum $\mathcal{L}_{\rm g.f.} + \mathcal{L}_{\rm ghost}$ under BRS transformations for renormalized fields is again guaranteed by the nilpotency $\mathsf{s}^2 f = 0$. This then implies $x = z = u$. Now we introduce the field renormalization constant for ghosts,

$$(c)_{\rm B} = Z_c c\,. \tag{2.42}$$

Since the first term in $\mathsf{s}f$ gives the ghost kinetic terms, it is natural to identify $y = Z_c$. This fixes all constants in $\mathsf{s}f$,

$$\begin{aligned} \mathsf{s}f \;=\;& Z_c \partial^\mu (\partial_\mu c + igZ_g[c, G_\mu]) + \\ &+ Z_c Z_g (1 + \delta v/v) g^2 v^2 \xi /2 (\hat{\phi}_0^\dagger c t^A \hat{\phi}_0 + \hat{\phi}_0^\dagger t^A c \hat{\phi}_0) t^A + \\ &+ Z_c Z_g g^2 v \xi/\sqrt{2} (\phi^\dagger c t^A \hat{\phi}_0 + \hat{\phi}_0^\dagger t^A c \phi) t^A\,. \end{aligned} \tag{2.43}$$

The counterterms for self-energies and the rules for vertices in the gauge fixing sector are also included in appendix A.

3. One loop divergences and renormalization constants

In this section we present our results of one loop divergences in 1PI Green's functions. Although we have exhausted all possibilities for each type of functions discussed below, it is not possible and also unnecessary to list all of them. Actually, a glance at the counterterms shows that all functions in the same type must have the same or similar divergent structure if divergences can be removed altogether with the renormalization constants introduced above. Instead, we demonstrate our results by typical examples. Since we are interested in the ultraviolet (UV) divergences, only diagrams which are apparently divergent by power counting are computed below. The complete Feynman diagrams are shown in appendix B. We thus will not touch upon the UV-IR mixing problem for exceptional momenta such as $\theta_{\mu\nu} p^\nu = 0$, etc. We work in the $\xi = 1$ gauge throughout for simplicity. Then the would-be Goldstone bosons and ghosts have the same masses as their corresponding gauge bosons.

3.1 Tadpole

The Feynman diagrams are shown in figure 1. The result is

$$\begin{aligned} iT \;=\;& +\lambda v \int \left[3D_k^\sigma + D_k^Z + 2D_k^W\right] + \\ &+ \frac{n}{2} g^2 v \int \left[D_k^W + D_k^Z\right] - \\ &- \frac{1}{2} g^2 v \int \left[D_k^W + D_k^Z\right], \end{aligned} \tag{3.1}$$

where

$$\int = \int \frac{d^n k}{(2\pi)^n}\,, \qquad D_p^j = \frac{1}{p^2 - m_j^2 + i\epsilon}\,. \tag{3.2}$$

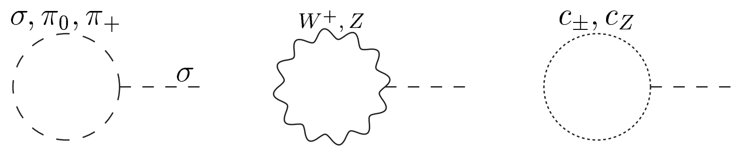

Figure 1: σ tadpole.

We work in $n = 4 - 2\epsilon$ dimensions to regularize the UV divergences. Note that there is no θ dependence in iT since we need at least two independent momenta for this. We obtain the divergent part as follows,

$$iT = i\Delta_\epsilon \left[6\lambda^2 + \lambda g^2 + \frac{9}{8} g^4 \right] v^3 , \qquad \Delta_\epsilon = \frac{1}{(4\pi)^2} \frac{1}{\epsilon} . \tag{3.3}$$

3.2 $\phi\phi$ self-energies and mixings

We have divergent $\sigma\sigma$, $\pi_0\pi_0$, $\pi_+\pi_-$ self-energies and the finite $\sigma\pi_0$ mixing. We compute the first and the last ones as examples. The Feynman diagrams for $\sigma\sigma$ are shown in figure 2. The result is

$$\begin{aligned} i\Sigma^{\sigma\sigma}(p) = &+\lambda^2 v^2 \int \left[c^2_{k\wedge p} \left(18 D^\sigma_k D^\sigma_{k+p} + 2 D^Z_k D^Z_{k+p} \right) + 4 D^W_k D^W_{k+p} \right] - \\ &-\frac{1}{2} g^2 \int (k+2p)^2 \left[s^2_{k\wedge p} D^Z_k D^\sigma_{k+p} + c^2_{k\wedge p} D^Z_k D^Z_{k+p} + D^W_k D^W_{k+p} \right] + \\ &+\frac{n}{4} g^4 v^2 \int \left[2 c^2_{k\wedge p} D^Z_k D^Z_{k+p} + D^W_k D^W_{k+p} \right] - \\ &-\frac{1}{8} g^4 v^2 \int \left[2 c^2_{k\wedge p} D^Z_k D^Z_{k+p} + D^W_k D^W_{k+p} \right] + \\ &+\lambda \int \left[(1 + 2 c^2_{k\wedge p}) D^\sigma_k + (2 - c_{k\wedge p}) D^Z_k + 2 D^W_k \right] + \\ &+\frac{n}{2} g^2 \int \left[D^W_k + D^Z_k \right] , \end{aligned} \tag{3.4}$$

where $s^m_{k\wedge p} = \sin^m(k \wedge p)$, $c^m_{k\wedge p} = \cos^m(k\wedge p)$ and $k \wedge p = \theta_{\mu\nu} k^\mu p^\nu/2$. Using $s^2_{k\wedge p} = (1 - c_{2k\wedge p})/2$ and $c^2_{k\wedge p} = (1 + c_{2k\wedge p})/2$ to separate the planar from the nonplanar part, and noting that the latter is finite due to the oscillating factor, we can isolate the UV divergences and obtain,

$$i\Sigma^{\sigma\sigma}(p) = i\Delta_\epsilon \left[-2g^2p^2 + 18\lambda^2 v^2 + \lambda g^2 v^2 + \frac{21}{8} g^4 v^2 \right] . \tag{3.5}$$

A similar calculation gives,

$$i\Sigma^{\pi_0\pi_0}(p) = i\Sigma^{\pi_+\pi_-}(p) = i\Delta_\epsilon \left[-2g^2p^2 + 6\lambda^2 v^2 + \lambda g^2 v^2 + \frac{9}{8} g^4 v^2 \right] . \tag{3.6}$$

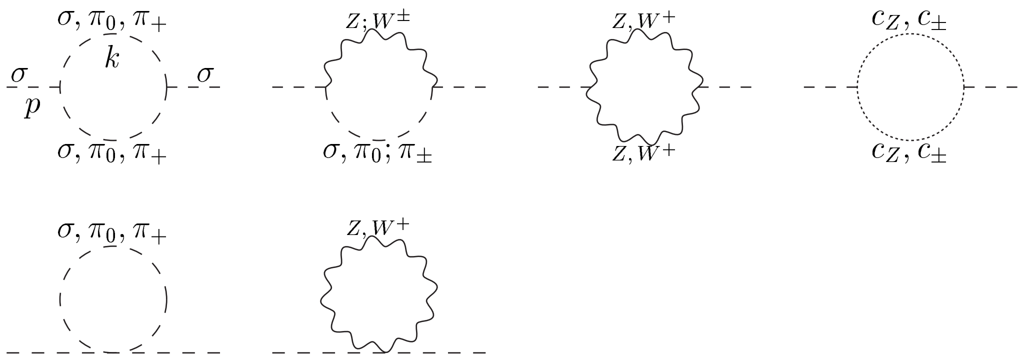

Figure 2: σ self-energy.

Figure 3: $\sigma\pi_0$ mixing.

Now we come to the $\sigma\pi_0$ mixing shown in figure 3. All of the neutral particle loops are found to be proportional to $s_{2k\wedge p}$ and thus finite. The $W^+\pi_+$ loop cancels exactly the $W^-\pi_-$ loop due to the sign flip in the σ couplings although they are separately divergent after cancellation of oscillating factors from the two vertices. The same happens for the $c_\pm$ loops due to the sign flip in the π_0 vertices. Thus we finally have a finite $\sigma\pi_0$ mixing.

3.3 GG self-energies and mixings

We have divergent AA, ZZ and W^+W^- self-energies and a finite AZ mixing. For the divergent one we take as an example the A self-energy whose diagrams are shown in figure 4. We find,

$$
\begin{aligned}
i\Sigma^{AA}_{\mu\nu}(p) = &+\frac{1}{2}g^2\int (2k+p)_\mu(2k+p)_\nu D^W_k D^W_{k+p} - \frac{1}{4}g^4v^2 g_{\mu\nu}\int D^W_k D^W_{k+p} + \\
&+\frac{1}{2}g^2\int P^2_{\mu\nu}\left[2s^2_{k\wedge p}D^A_k D^A_{k+p} + D^W_k D^W_{k+p}\right] -
\end{aligned}
$$

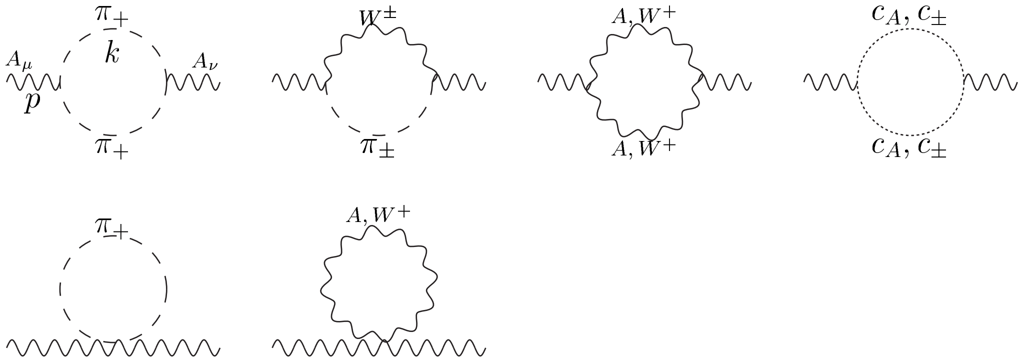

Figure 4: A self-energy.

$$
\begin{aligned}
&-g^2 \int (k+p)_\mu k_\nu \left[2 s^2_{k\wedge p} D^A_k D^A_{k+p} + D^W_k D^W_{k+p}\right] - \\
&-g^2 g_{\mu\nu} \int D^W_k + g^2 (1-n) g_{\mu\nu} \int \left[2 s^2_{k\wedge p} D^A_k + D^W_k\right] \\
= \; & i\Delta_\epsilon \frac{19}{6} g^2 (p^2 g_{\mu\nu} - p_\mu p_\nu)\,,
\end{aligned}
\tag{3.7}
$$

where $P^2_{\mu\nu} = P_{\alpha\beta\mu}(k, -k-p, p) P^{\alpha\beta}{}_\nu(-k, k+p, -p)$ with $P_{\alpha\beta\gamma}(k_1, k_2, k_3)$ given in appendix A. Identical results are obtained for the other two self-energies, especially there are no divergences proportional to $m^2_{W,Z}$.

The Feynman diagrams for the AZ mixing are shown in figure 5. Each diagram is finite due to an oscillating factor $\exp(i2k \wedge p)$. This occurs because the A and Z couplings to the same charged particles have opposite phases which become the same when the charges are reversed in one of the couplings.

3.4 $G\phi$ mixings

We have divergent $Z\pi_0$ and $W^\pm\pi_\mp$ mixings while $A\sigma$, $A\pi_0$ and $Z\sigma$ must be finite. The diagrams for $W^+\pi_-$ are shown in figure 6 where p denotes the incoming momentum of W^+. We obtain,

$$
\begin{aligned}
i\Sigma^{W^+\pi_-}_\mu(p) = \; & -gv\lambda \int (2k+p)_\mu D^\sigma_k D^W_{k+p} \\
& -\frac{1}{4} g^3 v \int (k+2p)_\mu \left[D^A_k D^W_{k+p} + D^W_k D^\sigma_{k+p}\right] \\
& +\frac{1}{4} g^3 v (1-n) \int (2k+p)_\mu \left[D^W_k D^A_{k+p} - D^W_k D^Z_{k+p}\right] \\
& +\frac{1}{4} g^3 v \int (k+p)_\mu \left[D^W_k D^A_{k+p} + D^Z_k D^W_{k+p}\right] \\
= \; & i\Delta_\epsilon (-g^2 m_W) p_\mu.
\end{aligned}
\tag{3.8}
$$

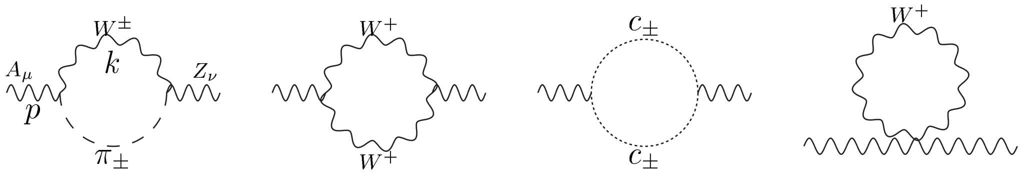

Figure 5: AZ mixing.

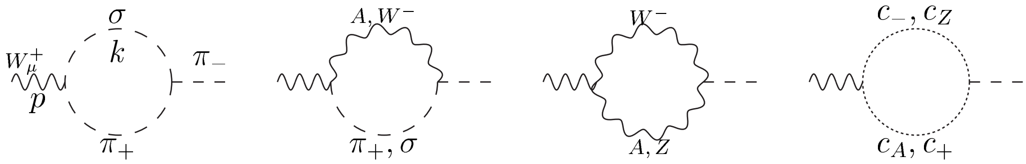

Figure 6: $W^+\pi_-$ mixing.

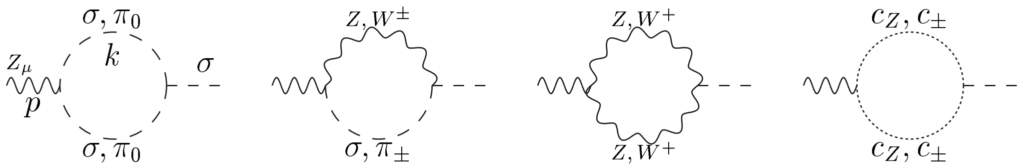

Figure 7: $Z\sigma$ mixing.

It is interesting to see how the $Z\sigma$ mixing shown in figure 7 becomes finite. The situation is similar to the case of the $\sigma\pi_0$ mixing. All neutral particle loops are proportional to $s_{2k\wedge p}$ and thus finite. The $W^{\pm}\pi_{\pm}$ loops exactly cancel themselves because of a sign flip in their couplings to σ. The same is true for the $c_{\pm}$ loops but this time the flip occurs in the Z couplings. Finally, the $W^{\pm}$ loop is finite since it is simply proportional to $(2k+p)_\mu$.

3.5 $c\bar{c}$ self-energies and mixings

The $c\bar{c}$ self-energies are the easiest to compute. We have divergent combinations $c_A\bar{c}_A$, $c_Z\bar{c}_Z$ and $c_{\pm}\bar{c}_{\mp}$, and the finite ones, $c_A\bar{c}_Z$ and $c_Z\bar{c}_A$. The result for the $c_+\bar{c}_-$

Figure 8: $c_+\bar{c}_-$ self-energy.

Figure 9: $c_A\bar{c}_Z$ mixing.

self-energy shown in figure 8 is,

$$
\begin{aligned}
i\Sigma^{c_+\bar{c}_-}(p) &= +\frac{1}{16}g^4v^2\int D_k^W\left[D_{k+p}^\sigma - D_{k+p}^Z\right] + \\
&\quad +\frac{1}{2}g^2\int k\cdot p\left[D_k^W\left(D_{k+p}^A + D_{k+p}^Z\right) + \left(D_k^A + D_k^Z\right)D_{k+p}^W\right] \\
&= i\Delta_\epsilon(-g^2p^2)\,. \qquad (3.9)
\end{aligned}
$$

Note that there is no divergence corresponding to the mass term. The $c_A\bar{c}_Z$ mixing shown in figure 9 is finite since each diagram contains an oscillating factor $\exp(\pm i2k\wedge p)$.

3.6 $\phi\phi\phi$ vertices

Now we start our computation of vertices with the trilinear scalar couplings. We have $\sigma\sigma\sigma$, $\sigma\pi_0\pi_0$ and $\sigma\pi_+\pi_-$ vertices which may be divergent, and $\sigma\sigma\pi_0$, $\pi_0\pi_0\pi_0$ and $\pi_0\pi_+\pi_-$ vertices which must be finite. We illustrate the computation with the last one of each type whose diagrams are shown in figures 10 and 11 respectively. The arrow inside the loop indicates the charge flow. p and $p_\pm$ are the incoming momenta of σ or π_0, and $\pi_\pm$.

Let us first consider the $\sigma\pi_+\pi_-$ vertex. Figure (a) contains the phase factor of $\exp(\pm i2p\wedge k)$ and is thus finite. The same is true for figures (c) and (g) which have a phase of $\exp(i2p\wedge k)$ respectively. The other diagrams are divergent. For the σ loop in figure (b), we have

$$
(b)_\sigma = -\frac{3}{2}\lambda g^2v\int O_k(k+2p_+)\cdot(k-2p_-)D_k^W D_{k+p_+}^\sigma D_{k-p_-}^\sigma\,. \qquad (3.10)
$$

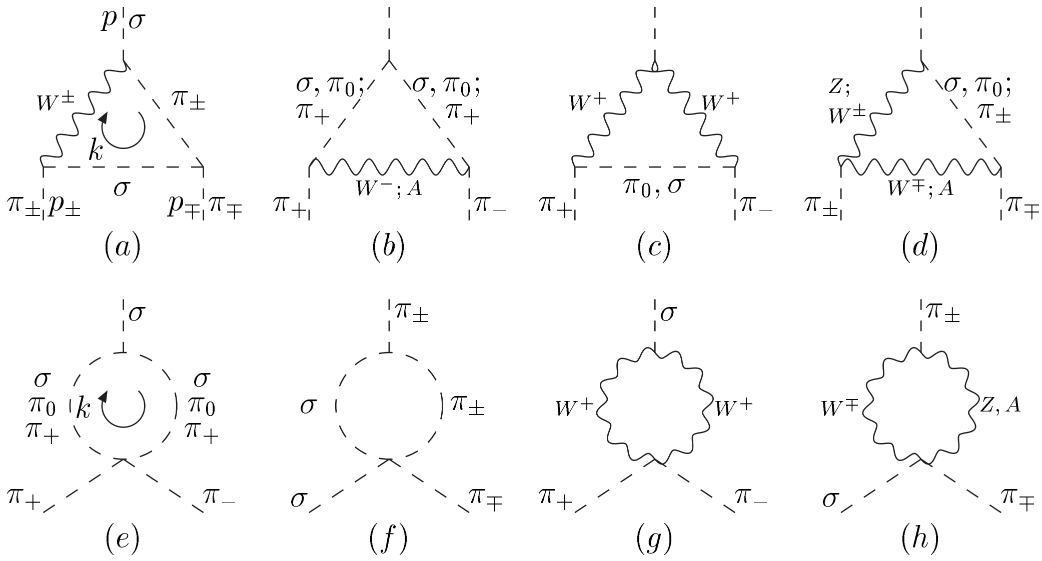

Figure 10: $\sigma\pi_+\pi_-$ vertex.

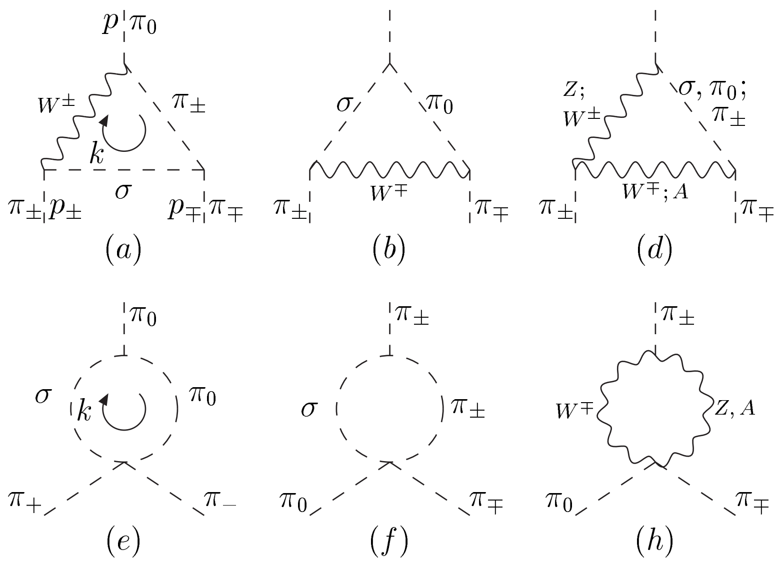

Figure 11: $\pi_0\pi_+\pi_-$ vertex.

Using

$$
\begin{aligned}
O_k &= c_{p\wedge(k+p_+)} \exp(ik \wedge p) \\
&= 1/2 \;\; \exp(ip_+ \wedge p_-) + \cdots
\end{aligned}
\tag{3.11}
$$

to isolate the non-k oscillating part, we obtain the divergence,

$$(b)_\sigma = -\frac{3}{4}\lambda g^2 v \exp(ip_+ \wedge p_-) i\Delta_\epsilon \,. \tag{3.12}$$

The other two are similarly computed with the sum,

$$(b) = \left[-\frac{3}{4} - \frac{1}{4} - 1\right] \lambda g^2 v \exp(ip_+ \wedge p_-) i\Delta_\epsilon \,. \tag{3.13}$$

The same trick applies to figure (d). Here the charge conjugated loops contribute the same. Using obvious notations, we have

$$(d) = 2 \cdot \left[-\frac{1}{16} - \frac{1}{16} - \frac{1}{8}\right] g^4 v \exp(ip_+ \wedge p_-) i\Delta_\epsilon \,. \tag{3.14}$$

Figures $(e), (f)$ and (h) are easier to compute with the result,

$$\begin{aligned}
(e) &= +(3+1+4)\lambda^2 v \exp(ip_+ \wedge p_-) i\Delta_\epsilon \,, \\
(f) &= +2 \cdot 2\lambda^2 v \exp(ip_+ \wedge p_-) i\Delta_\epsilon \,, \\
(h) &= +2 \cdot (1/2 + 1/2) g^4 v \exp(ip_+ \wedge p_-) i\Delta_\epsilon \,.
\end{aligned} \tag{3.15}$$

The divergence in the $\sigma\pi_+\pi_-$ vertex is then

$$iV^{\sigma\pi_+\pi_-}(p, p_+, p_-) = i\Delta_\epsilon \exp(ip_+ \wedge p_-) \left[\frac{3}{2} g^4 - 2\lambda g^2 + 12\lambda^2\right] v \,. \tag{3.16}$$

Let us compare the contributions to the $\pi_0\pi_+\pi_-$ vertex shown in figure 11 with the above one. First, the analogs of figures 10(c) and (g) are missing due to lack of the $\pi_0 W^+ W^-$ vertex. Second, (a) is similarly finite. (e) is also finite because of the factor $s_{2k\wedge p}$. Third, the others always come in a charge conjugated pair whose divergences cancel each other. We take (b) as an example. The product of the two $G\phi\phi$ vertices in the $W^\mp$ loop is

$$\mp\frac{i}{4} g^2 (k + 2p_+) \cdot (k - 2p_-) \,, \tag{3.17}$$

while remaining factors are essentially the same for the consideration of divergence so that the divergences are cancelled between the $W^\mp$ loops.

3.7 $G\phi\phi$ vertices

We have vertices $A\pi_+\pi_-$, $Z\sigma\sigma$, $Z\sigma\pi_0$, $Z\pi_0\pi_0$, $W^\pm\pi_\mp\sigma$ and $W^\pm\pi_\mp\pi_0$ which may be divergent, and vertices $A\sigma\sigma$, $A\sigma\pi_0$, $A\pi_0\pi_0$ and $Z\pi_+\pi_-$ which must be finite. For illustration we compute the first one of each type whose diagrams are given in figures 12 and 13.

Figure 12(a) has a phase $\exp(i2k \wedge p)$ and is finite. (d) is also finite: while the A loop involves $s_{2k\wedge p}$, the W loop is proportional to $(k + 2p)_\mu$ and thus vanishes.

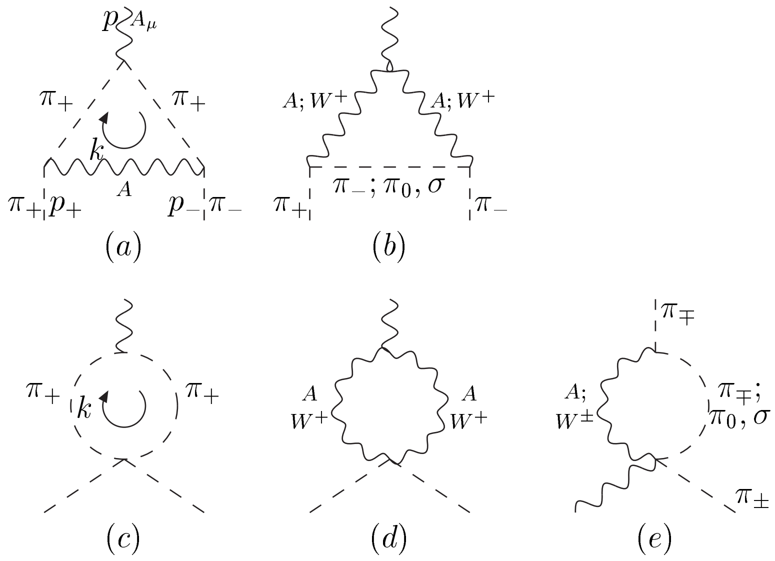

Figure 12: $A_\mu \pi_+ \pi_-$ vertex.

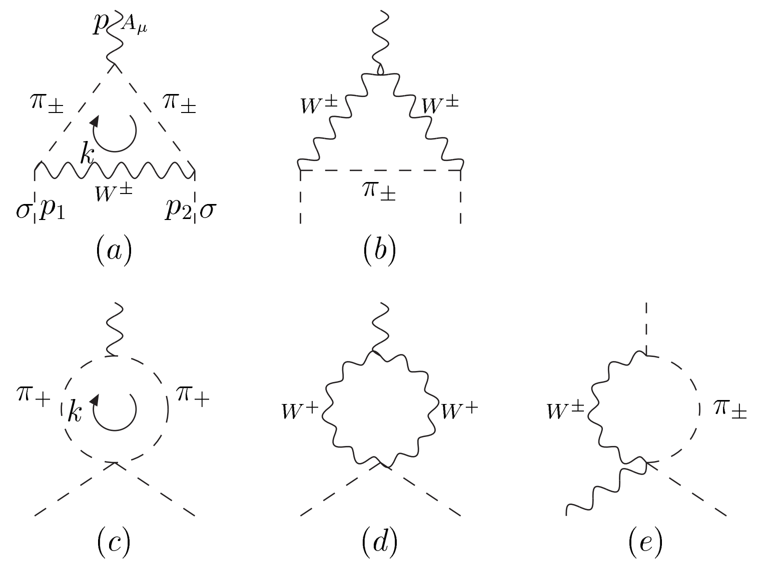

Figure 13: $A_\mu \sigma\sigma$ vertex.

For similar reasons (c) is finite too. So we only need to calculate (b) and (e). For example,

$$
\begin{aligned}
(b)_{\pi_-} &= -\frac{i}{\sqrt{2}} g^3 \int O_k P_\mu(k) D_k^W D_{k+p_+}^A D_{k-p_-}^A \,, \\
O_k &= \sin(p_+ \wedge p_- + p \wedge k) \exp(ip \wedge k) \\
&= i/2 \ \exp(-ip_+ \wedge p_-) + \cdots \,, \\
P_\mu(k) &= P_{\mu\alpha\beta}(p, k + p_+, p_- - k)(k - p_+)^\alpha (k + p_-)^\beta \\
&= 2[k^2(p_+ - p_-)_\mu - k_\mu k \cdot (p_+ - p_-)] + \cdots \,,
\end{aligned} \tag{3.18}
$$

where we have ignored terms which will be finite. The divergence is

$$
(b)_{\pi_-} = i\Delta_\epsilon \frac{3}{4\sqrt{2}} g^3 \exp(-ip_+ \wedge p_-)(p_+ - p_-)_\mu \,. \tag{3.19}
$$

Together with π_0 and σ loops, we have

$$
(b) = i\Delta_\epsilon \frac{3}{4\sqrt{2}} g^3 \left[1 + \frac{1}{2} + \frac{1}{2}\right] \exp(-ip_+ \wedge p_-)(p_+ - p_-)_\mu \,. \tag{3.20}
$$

Figure (e) comes in conjugated pairs and is simpler to compute. For example,

$$
\begin{aligned}
(e)_{\pi_-} &= +\frac{1}{\sqrt{2}} g^3 \int O_k (k + 2p_-)_\mu D_k^A D_{k+p_-}^W \\
&= +i\Delta_\epsilon \frac{3}{4\sqrt{2}} g^3 \exp(-ip_+ \wedge p_-) p_{-\mu} \,,
\end{aligned} \tag{3.21}
$$

where

$$
\begin{aligned}
O_k &= c_{p\wedge k} \exp(ip \wedge k - ip_+ \wedge p_-) \\
&= 1/2 \ \exp(-ip_+ \wedge p_-) + \cdots \,.
\end{aligned} \tag{3.22}
$$

The conjugated π_+ loop is then obtained by $p_{-\mu} \to -p_{+\mu}$. Including the other two pairs of loops, we have

$$
(e) = -i\Delta_\epsilon \frac{3}{4\sqrt{2}} g^3 \left[1 + \frac{1}{2} + \frac{1}{2}\right] \exp(-ip_+ \wedge p_-)(p_+ - p_-)_\mu \,, \tag{3.23}
$$

which cancels figure (b) exactly so that the one loop contribution to the $A_\mu \pi_+ \pi_-$ vertex is finite.

In contrast, the one loop contribution to the $A_\mu \sigma\sigma$ vertex is finite because each individual diagram involves the same oscillatory phase $\exp(i2k \wedge p)$.

3.8 $GG\phi$ vertices

We have possibly divergent vertices of $W^\pm A \pi_\mp$, $W^\pm Z \pi_\mp$, $ZZ\sigma$ and $W^+W^-\sigma$ while the vertices $AA\sigma$, $AA\pi_0$, $ZZ\pi_0$, $AZ\sigma$, $AZ\pi_0$ and $W^+W^-\pi_0$ must be finite. We show our calculation by the last one of each type.

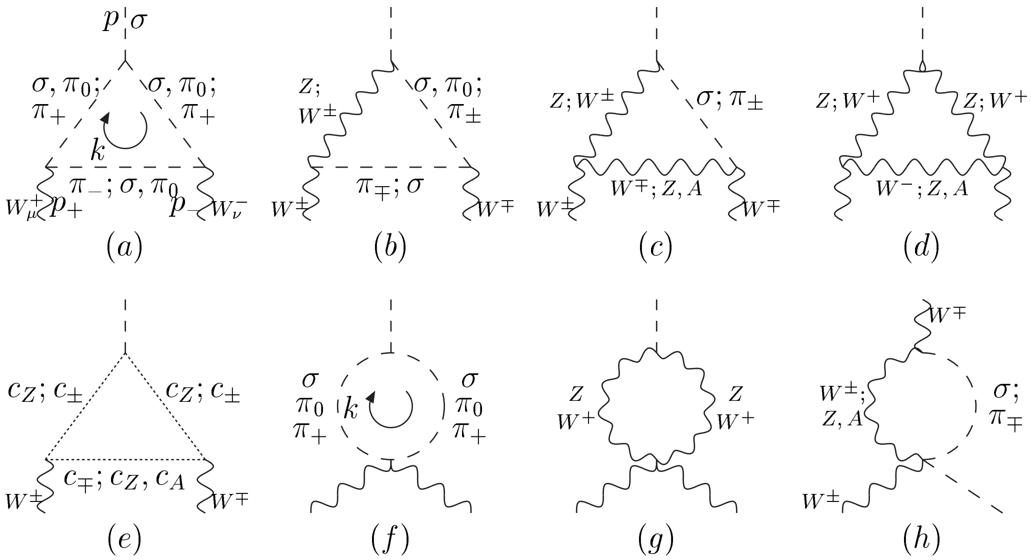

Figure 14: $W^+_\mu W^-_\nu \sigma$ vertex.

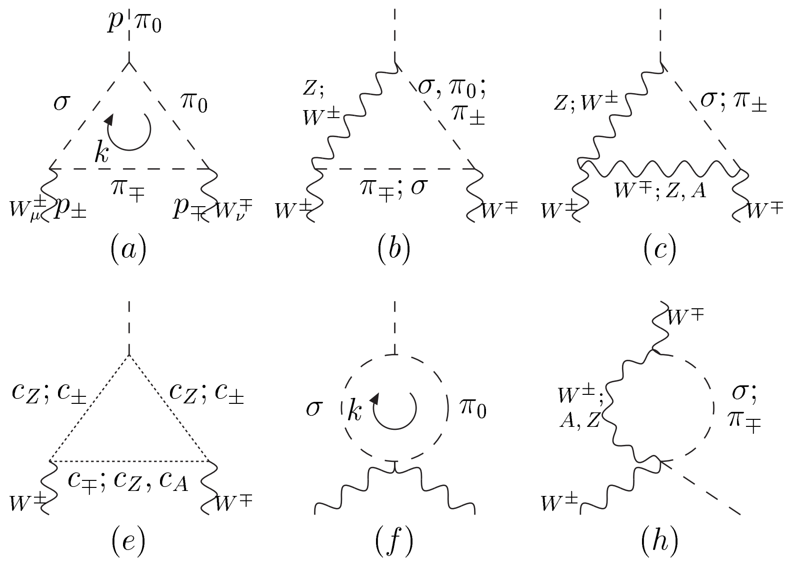

Figure 15: $W^+_\mu W^-_\nu \pi_0$ vertex.

The one loop diagrams for the $W^+_\mu W^-_\nu \sigma$ vertex are shown in figure 14. Now it should be relatively easy to compute figure (a), so we write down its divergence directly,

$$(a) = i\Delta_\epsilon \lambda g^2 v g_{\mu\nu} \exp(ip_+ \wedge p_-) \left[\frac{3}{4} + \frac{1}{4} + 0 + 0\right]. \tag{3.24}$$

The $W^\pm$ loops in (b) are made finite by the phase $\exp(i2p \wedge k)$ while the divergences in the $Z\sigma\pi_\mp$ loops are cancelled by those of the $Z\pi_0\pi_\mp$ loops due to a sign flip in the relevant part of products of the $G\phi\phi$ vertices. The σW^- loop of (c) is,

$$\begin{aligned}
(c)_{\sigma W^-} &= +\frac{i}{4} g^4 v \int O_k P_{\mu\nu} D_k^W D_{k+p_+}^Z D_{k-p_-}^\sigma \\
&= i\Delta_\epsilon \frac{3}{32} g^4 v g_{\mu\nu} \exp(ip_+ \wedge p_-) + \cdots ,
\end{aligned} \tag{3.25}$$

where

$$\begin{aligned}
O_k &= \sin(p \wedge (p_- - k)) \exp(ik \wedge p) \\
&= i/2 \exp(ip_+ \wedge p_-) + \cdots , \\
P_{\mu\nu} &= (p - p_- + k)^\beta P_{\beta\mu\nu}(-k - p_+, p_+, k) \\
&= (-k^2 g_{\mu\nu} + k_\mu k_\nu) + \cdots .
\end{aligned} \tag{3.26}$$

The remaining $\pi_\pm$ loops are similarly computed. Including charge conjugated loops, we obtain,

$$(c) = +i\Delta_\epsilon g^4 v g_{\mu\nu} \exp(ip_+ \wedge p_-)\ 2 \cdot \left[\frac{3}{32} + 0 + \frac{3}{32}\right]. \tag{3.27}$$

Figure (d) is slightly more complicated. We take the ZZW^- loop as an example,

$$\begin{aligned}
(d)_{ZZW^-} &= +\frac{1}{2} g^4 v \int O_k P_{\mu\nu} D_k^W D_{k+p_+}^Z D_{k-p_-}^Z \\
&= +i\Delta_\epsilon \frac{9}{8} g^4 v g_{\mu\nu} \exp(ip_+ \wedge p_-) + \cdots ,
\end{aligned} \tag{3.28}$$

where

$$\begin{aligned}
O_k &= \cos(p_+ \wedge p_- + p \wedge k) \exp(ik \wedge p) \\
&= 1/2 \exp(ip_+ \wedge p_-) + \cdots , \\
P_{\mu\nu} &= P_{\alpha\mu}{}^\beta(-k - p_+, p_+, k) P^\alpha{}_{\beta\nu}(k - p_-, -k, p_-) \\
&= (2k^2 g_{\mu\nu} + 10 k_\mu k_\nu) + \cdots .
\end{aligned} \tag{3.29}$$

Including the other two contributions, we have

$$(d) = +i\Delta_\epsilon g^4 v g_{\mu\nu} \exp(ip_+ \wedge p_-) \left[\frac{9}{8} + 0 + \frac{9}{8}\right]. \tag{3.30}$$

The calculation of figure (e) is similar to (a), and figures (f)–(h) are the easiest of all, with the results,

$$(e) = -i\Delta_\epsilon g^4 v g_{\mu\nu} \exp(ip_+ \wedge p_-)\ 2 \cdot \left[\frac{1}{32} + 0 + \frac{1}{32}\right],$$

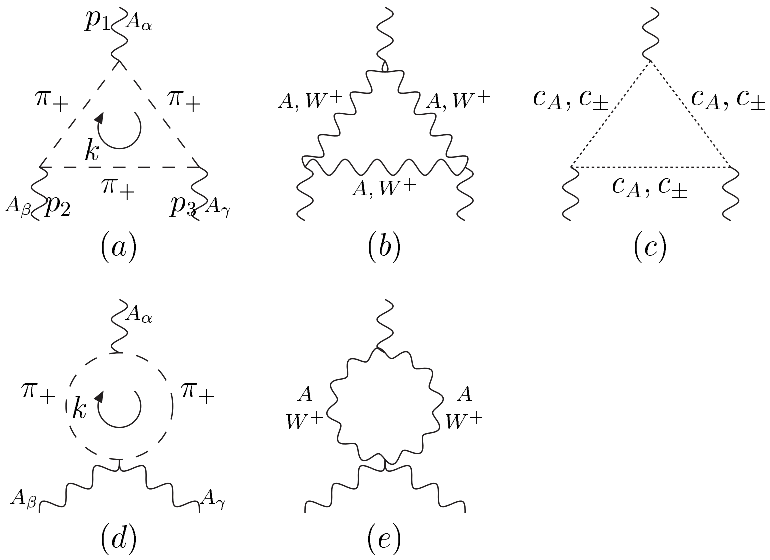

Figure 16: $A_\alpha A_\beta A_\gamma$ vertex.

$$\begin{aligned}
(f) &= -i\Delta_\epsilon \lambda g^2 v g_{\mu\nu} \exp(ip_+ \wedge p_-) \left[\frac{3}{4} + \frac{1}{4} + 0\right], \\
(g) &= -i\Delta_\epsilon g^4 v g_{\mu\nu} \exp(ip_+ \wedge p_-) \left[\frac{3}{4} + \frac{3}{4}\right], \\
(h) &= -i\Delta_\epsilon g^4 v g_{\mu\nu} \exp(ip_+ \wedge p_-)\ 2 \cdot \left[\frac{1}{8} + 0 + \frac{1}{8}\right].
\end{aligned} \tag{3.31}$$

In total,

$$iV_{\mu\nu}^{W^+W^-\sigma}(p_+, p_-, p) = +i\Delta_\epsilon \frac{1}{2} g^4 v g_{\mu\nu} \exp(ip_+ \wedge p_-)\,. \tag{3.32}$$

For the one loop $W_\mu^+ W_\nu^- \pi_0$ vertex shown in figure 15 we briefly indicate how a finite result is achived. The following diagrams are separately finite due to an oscillatory factor $\exp(i2p \wedge k)$ or $s_{2p\wedge k}$: $W^\pm$ loops in (b), $ZW^\pm\pi_\pm$ in (c), $c_Z c_\pm c_\pm$ in (e), $Z\pi_\mp$ in (h) and figure (f). The others cancel their divergences between conjugated diagrams.

3.9 GGG vertices

This is the most complicated part of the calculation performed in this section because of the involvement of GGG and $GGGG$ types of vertices. The one loop contributions to the vertices AAA, ZZZ, AW^+W^- and ZW^+W^- are generally divergent while those of AZZ and AAZ must be finite. Again we present our calculation using the first one of each type as examples.

Note that except for the A loop in figure 16(b) there is an additional permutated contribution for each case in (a)–(c) and there are two additional ones for (d)–(e). We first note that (d) vanishes identically. This is because the only difference from the analog in ordinary scalar QED is the appearance of the factor $c_{p_2\wedge p_3}$. For figure (a) we have,

$$\begin{aligned}(a)^{123} &= -\frac{g^3}{2\sqrt{2}}\exp(ip_3\wedge p_2)\int D^W_k D^W_{k+p_2} D^W_{k-p_3}\times\\ &\quad\times(2k+p_2-p_3)_\alpha(2k+p_2)_\beta(2k-p_3)_\gamma\\ &= -i\Delta_\epsilon\frac{g^3}{6\sqrt{2}}\exp(ip_3\wedge p_2)P_{\alpha\beta\gamma}(p_1,p_2,p_3)+\cdots\,.\end{aligned}\tag{3.33}$$

Summing with the permutated one gives

$$\begin{aligned}(a) &= (a)^{123}+(a)^{132}\\ &= -\Delta_\epsilon\frac{g^3}{3\sqrt{2}}\sin(p_2\wedge p_3)P_{\alpha\beta\gamma}(p_1,p_2,p_3)+\cdots\,.\end{aligned}\tag{3.34}$$

The A loop in (b) is

$$\begin{aligned}(b)_A &= -i2\sqrt{2}g^3\int O_k P^t_{\alpha\beta\gamma} D^A_k D^A_{k+p_2} D^A_{k-p_3}\\ &= -\Delta_\epsilon\frac{13g^3}{4\sqrt{2}}\sin(p_1\wedge p_2)P_{\alpha\beta\gamma}(p_1,p_2,p_3)+\cdots\,,\end{aligned}\tag{3.35}$$

where

$$\begin{aligned}O_k &= \sin(p_1\wedge(k+p_2))\sin(p_2\wedge k)\sin(k\wedge p_3)\\ &= 1/4\ \sin(p_1\wedge p_2)+\cdots,\\ P^t_{\alpha\beta\gamma} &= P_\alpha{}^{\rho\tau}(p_1,k+p_2,p_3-k)P_{\beta\sigma\rho}(p_2,k,-k-p_2)\times\\ &\quad\times P_\gamma{}^\sigma{}_\tau(p_3,-k,k-p_3)\,.\end{aligned}\tag{3.36}$$

The W^+ loop is similarly computed. Including its permutation, we have the divergence

$$\begin{aligned}(b)_W &= (b)^{123}_W+(b)^{132}_W\\ &= -i\Delta_\epsilon\frac{13g^3}{8\sqrt{2}}\left[\exp(ip_3\wedge p_2)-\exp(ip_2\wedge p_3)\right]P_{\alpha\beta\gamma}(p_1,p_2,p_3)\\ &= -\Delta_\epsilon\frac{13g^3}{4\sqrt{2}}\sin(p_2\wedge p_3)P_{\alpha\beta\gamma}(p_1,p_2,p_3)\,.\end{aligned}\tag{3.37}$$

Figure (c) is essentially similar to (a) but we must be careful with the tensor $P_{\alpha\beta\gamma}(p_1,p_2,p_3)$. Using its properties given in appendix A, we have

$$(c)_A = (c)^{123}_A+(c)^{132}_A$$

$$= -\Delta_\epsilon \frac{g^3}{12\sqrt{2}} \sin(p_2 \wedge p_3) \left[P_{\gamma\alpha\beta}(p_1, p_2, p_3) - P_{\beta\alpha\gamma}(p_1, p_3, p_2)\right]$$
$$= +\Delta_\epsilon \frac{g^3}{12\sqrt{2}} \sin(p_2 \wedge p_3) P_{\alpha\beta\gamma}(p_1, p_2, p_3)\,. \tag{3.38}$$

The separate contribution of $c_\pm$ does not have the same structure as the counterterm,

$$\begin{aligned}(c)_\pm &= (c)_\pm^{123} + (c)_\pm^{132} \\ &= \mp i\Delta_\epsilon \frac{g^3}{24\sqrt{2}} \left[e^{\pm i p_3 \wedge p_2} P_{\gamma\alpha\beta}(p_1, p_2, p_3) + e^{\pm i p_2 \wedge p_3} P_{\beta\alpha\gamma}(p_1, p_3, p_2)\right],\end{aligned} \tag{3.39}$$

but their sum has,

$$\begin{aligned}(c) &= (c)_+ + (c)_- \\ &= +\Delta_\epsilon \frac{g^3}{12\sqrt{2}} \sin(p_2 \wedge p_3) P_{\alpha\beta\gamma}(p_1, p_2, p_3)\,.\end{aligned} \tag{3.40}$$

Now we compute the A loop in figure (e),

$$(e)_A^1 = -i\sqrt{2} g^3 \int T_{\alpha\beta\gamma}(k, p_i) D_k^A D_{k+p_1}^A\,, \tag{3.41}$$

where

$$\begin{aligned}T_{\alpha\beta\gamma}(k, p_i) = &\; P_\alpha{}^{\mu\nu}(p_1, k, -k - p_1) s_{p_1 \wedge k} \times \\ &\times [(g_{\mu\beta} g_{\nu\gamma} - g_{\mu\gamma} g_{\nu\beta}) s_{p_2 \wedge p_3} s_{p_1 \wedge k} + \\ &+ (g_{\mu\gamma} g_{\nu\beta} - g_{\mu\nu} g_{\beta\gamma}) s_{p_2 \wedge k} s_{p_3 \wedge (k+p_1)} + \\ &+ (g_{\mu\nu} g_{\beta\gamma} - g_{\mu\beta} g_{\nu\gamma}) s_{k \wedge p_3} s_{p_2 \wedge (k+p_1)}]\,.\end{aligned} \tag{3.42}$$

Isolating the non-k oscillatory part as follows,

$$\begin{aligned}s_{p_1 \wedge k} s_{p_2 \wedge p_3} s_{p_1 \wedge k} &= +1/2 \; s_{p_2 \wedge p_3} + \cdots\,, \\ s_{p_1 \wedge k} s_{p_2 \wedge k} s_{p_3 \wedge (k+p_1)} &= -1/4 \; s_{p_3 \wedge p_1} + \cdots\,, \\ s_{p_1 \wedge k} s_{k \wedge p_3} s_{p_2 \wedge (k+p_1)} &= -1/4 \; s_{p_1 \wedge p_2} + \cdots\,,\end{aligned} \tag{3.43}$$

and doing loop integration by using the explicit form of $P_\alpha{}^{\mu\nu}$, we arrive at

$$(e)_A^1 = +\Delta_\epsilon \frac{9}{2\sqrt{2}} g^3 \sin(p_2 \wedge p_3)(p_{1\gamma} g_{\alpha\beta} - p_{1\beta} g_{\gamma\alpha}) + \cdots\,. \tag{3.44}$$

Including all permutations, we recover the correct structure,

$$\begin{aligned}(e)_A &= (e)_A^1 + (e)_A^2 + (e)_A^3 \\ &= +\Delta_\epsilon \frac{9}{2\sqrt{2}} g^3 \sin(p_2 \wedge p_3) P_{\alpha\beta\gamma}(p_1, p_2, p_3) + \cdots\,.\end{aligned} \tag{3.45}$$

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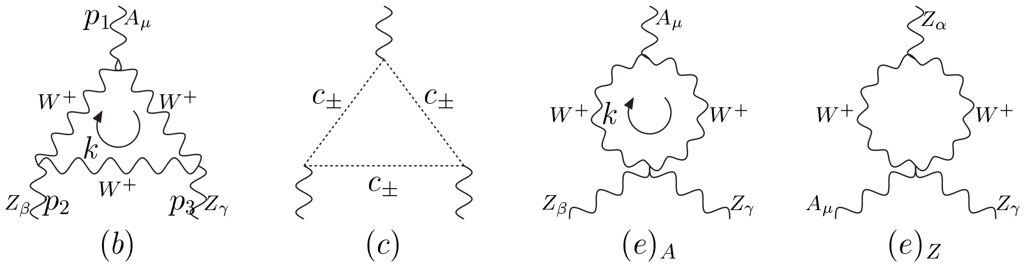

Figure 17: $A_\mu Z_\alpha Z_\beta$ vertex.

The W^+ loop is simpler and turns out to have the same divergence as the A loop. In summary, the divergence in the vertex is,

$$iV^{AAA}_{\alpha\beta\gamma}(p_1,p_2,p_3) = +\Delta_\epsilon \frac{7}{3\sqrt{2}} g^3 \sin(p_2\wedge p_3) P_{\alpha\beta\gamma}(p_1,p_2,p_3)\,. \tag{3.46}$$

For the $A_\mu Z_\alpha Z_\beta$ vertex shown in figure 17 we just comment that there are no analogs of figures 16(a) and (d) and that the remaining diagrams are made finite by the phase $\exp(i2k\wedge p_i)$.

3.10 $\phi c\bar{c}$ vertices

We have the following list of possibly divergent vertices $\sigma c_Z\bar{c}_Z$, $\sigma c_\pm\bar{c}_\mp$, $\pi_0 c_Z\bar{c}_Z$, $\pi_0 c_\pm\bar{c}_\mp$, $\pi_\pm c_\mp\bar{c}_Z$ and $\pi_\pm c_A\bar{c}_\mp$, and the following one which must be finite, $\sigma c_A\bar{c}_A$, $\sigma c_A\bar{c}_Z$, $\sigma c_Z\bar{c}_A$, $\pi_0 c_A\bar{c}_A$, $\pi_0 c_A\bar{c}_Z$, $\pi_0 c_Z\bar{c}_A$, $\pi_\pm c_\mp\bar{c}_A$ and $\pi_\pm c_Z\bar{c}_\mp$.

Let us take a sample calculation of the $\sigma c_Z\bar{c}_Z$ vertex shown in figure 18. The σ loop is

$$\begin{aligned}(\sigma) &= +\frac{1}{2}g^4 v \int O_k k\cdot(k-p+\bar{q}) D^Z_k D^Z_{k-q} D^\sigma_{k+\bar{q}} \\ &= -i\Delta_\epsilon \frac{1}{8} g^4 v \cos(\bar{q}\wedge q) + \cdots,\end{aligned} \tag{3.47}$$

where we have used

$$\begin{aligned}O_k &= \sin(p\wedge(\bar{q}+k))\sin(q\wedge k)\cos(\bar{q}\wedge k) \\ &= -1/4\ \cos(\bar{q}\wedge q) + \cdots\,.\end{aligned} \tag{3.48}$$

The π_0 loop contributes the same, and the sum of the $\pi_\pm$ loops is

$$(\pi_+) + (\pi_-) = -i\Delta_\epsilon \frac{1}{8} g^4 v\,[\exp(i\bar{q}\wedge q) + \exp(iq\wedge\bar{q})] + \cdots\,. \tag{3.49}$$

The total divergence is then,

$$iV^{\sigma c_Z \bar{c}_Z}(p,q,\bar{q}) = -i\Delta_\epsilon \frac{1}{2} g^4 v \cos(\bar{q}\wedge q)\,. \tag{3.50}$$

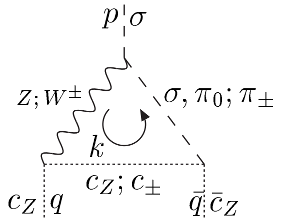

Figure 18: $\sigma c_Z \bar{c}_Z$ vertex.

As for the finite vertices, there are no apparently divergent diagrams at all for the vertices $\sigma c_A \bar{c}_A$, $\sigma c_Z \bar{c}_A$, $\pi_0 c_A \bar{c}_A$, $\pi_0 c_Z \bar{c}_A$ and $\pi_\pm c_\mp \bar{c}_A$. Figure 19 is an example of the remaining ones which however is finite due to the appearance of the phase $\exp(i2q \wedge k)$.

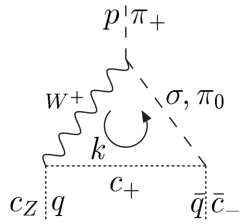

Figure 19: $\pi_+ c_Z \bar{c}_-$ vertex.

3.11 $Gc\bar{c}$ vertices

This is the last group of vertices computed in this section. We have generally divergent vertices $Ac_A\bar{c}_A$, $Ac_\pm\bar{c}_\mp$, $Zc_Z\bar{c}_Z$, $Zc_\pm\bar{c}_\mp$, $W^\pm c_\mp \bar{c}_A$, $W^\pm c_\mp \bar{c}_Z$, $W^\pm c_A \bar{c}_\mp$ and $W^\pm c_Z \bar{c}_\mp$, and finite ones $Ac_Z\bar{c}_Z$, $Ac_A\bar{c}_Z$, $Ac_Z\bar{c}_A$, $Zc_A\bar{c}_A$, $Zc_A\bar{c}_Z$ and $Zc_Z\bar{c}_A$. We illustrate our calculation by the examples shown in figures 20 and 21.

The c_A loop in figure 20(a) is

$$(a)_{c_A} = -\frac{i}{\sqrt{2}} g^3 \int O_k P_\mu D^A_k D^A_{k+\bar{q}} D^W_{k-q} \tag{3.51}$$

$$= -i\Delta_\epsilon \frac{3}{8\sqrt{2}} g^3 \exp(i\bar{q} \wedge q) \bar{q}_\mu + \cdots , \tag{3.52}$$

where we have used the following

$$\begin{aligned} O_k &= \sin(\bar{q} \wedge k) \exp(iq \wedge k) \exp(ip \wedge (k-q)) \\ &= -i/2 \ \exp(i\bar{q} \wedge q) + \cdots , \\ P_\mu &= P_{\alpha\mu\beta}(k+\bar{q}, p, q-k) \bar{q}^\alpha k^\beta \\ &= (k^2 \bar{q}_\mu - k \cdot \bar{q} k_\mu) + \cdots , \end{aligned} \tag{3.53}$$

to single out the divergence. The A exchange in the c_- loop is finite while the Z exchange contributes the same as the c_A loop. Figure (b) is simpler, so we write down the result directly,

$$(b) = -i\Delta_\epsilon \frac{1}{8\sqrt{2}} g^3 \exp(i\bar{q} \wedge q) \bar{q}_\mu (0+1+1) . \tag{3.54}$$

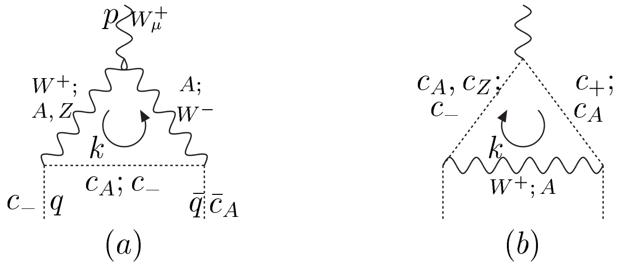

Figure 20: $W_\mu^+ c_- \bar{c}_A$ vertex.

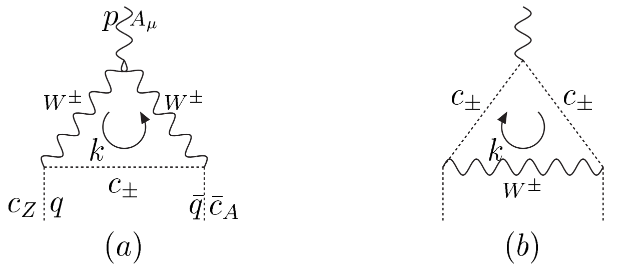

Figure 21: $A_\mu c_Z \bar{c}_A$ vertex.

The final result is,

$$iV^{W^+ c_- \bar{c}_A}(p, q, \bar{q}) = -i\Delta_\epsilon \frac{1}{\sqrt{2}} g^3 \exp(i\bar{q} \wedge q)\bar{q}_\mu \,. \tag{3.55}$$

One sentence suffices for the vertex shown in figure 21: all loops are driven finite by a phase of $\exp(\pm i2k \wedge p)$ or $\exp(\pm i2q \wedge k)$.

3.12 Renormalization constants

We have finished computing divergences in one loop 1PI functions in previous subsections. Using the counterterms described in appendix A, we determine the renormalization constants in the MS scheme as follows,

$$\delta Z_G = \Delta_\epsilon \frac{19}{6} g^2 \,, \qquad \delta Z_\phi = \Delta_\epsilon 2g^2 \,, \qquad \delta Z_c = \Delta_\epsilon g^2 \,,$$
$$\delta Z_g = -\Delta_\epsilon 2g^2 \,, \qquad \frac{\delta v}{v} = \Delta_\epsilon g^2 \,,$$

$$\lambda \delta Z_\lambda = \Delta_\epsilon \left[6\lambda^2 - 2\lambda g^2 + \frac{3}{4} g^4 \right],$$
$$\lambda \frac{\delta \mu^2}{\mu^2} = -\Delta_\epsilon \left[\lambda g^2 + \frac{3}{8} g^4 \right]. \tag{3.56}$$

These will be sufficient to remove all of UV divergences in Green's functions at one loop level. For example, the simple mass relation $m_Z^2 = 2m_W^2$ is preserved by the divergent parts of their respective counterterms which provides a reassuring check of the consistency of the model at one loop level.

4. Summary

A potential obstacle in attemps to construct consistent models of gauge interactions on NC spacetime is whether the renormalizability property can be still maintained or the concept of renormalization itself has to be modified. While there is no general proof of this so far, we can still get some feeling and confidence from explicit analyses. It has been checked that the exact U(1) and U(N) gauge theories and the spontaneously broken U(1) gauge theory can be consistently renormalized at one loop order. In this work we tried to fill the gap by including the spontaneously broken U(N) case. We emphasize that this latter case is distinct from the former ones. Since the gauge symmetry is partly broken we simultaneously have massive and massless gauge bosons which also makes the model closer to the standard model of electroweak interactions. The interactions and masses of these particels are simply related because they are in the same multiplet before symmetry breaking occurs. It is not clear from previous studies whether these relations can still be accommodated at the quantum level on NC spacetime. This is a nontrivial problem, considering the difficulties already met with spontaneous breaking of global symmetries. Our explicit analysis shows however that this is indeed possible; just as we see in the usual gauge theories, with the same limited number of renormalization constants we can remove the UV divergences for both exact and spontaneously broken non-abelian gauge theories on NC spacetime however complicated the latter case could be. This positive result supports the points of view that it is worthwhile to pursue further in this direction.

Although our explicit analysis is confined to the U(2) case for both physical and technical reasons, it seems reasonable to expect that our affirmative result also applies to the general case since the most important feature has already appeared in the U(2) case as discussed above, namely, the relations in interactions and masses among massless and massive gauge bosons as dictated by the group structure.

We have not discussed in this work the four point 1PI functions due to technical reasons. A glimpse at Feynman rules makes it clear that the model considered here is already much more complicated than the complete standard model of electroweak interactions. This happens in two related ways; we have more types (almost all imaginable types) of interactions and the interactions themselves become complicated due to the involvement of momentum-dependent phase factors. It is amazing to see in the previous section how different sectors of the model conspire to bring about a consistent result of UV divergences so that the above complications would not spoil the renormalizability of the model. It would not be surprising that the same magic also occurs in four point functions since they are in a sense related to the lower ones by the group structure and the same noncommutative relations. We also have not included fermions. We should expect no problems with vector-like fermions, but it will be rather delicate if chiral fermions are involved due to the danger of anomalies. All this deserves a separate work to which we hope to return soon.

Acknowledgments

I am grateful to H. Grosse and K. Sibold for many helpful discussions and also to K. Sibold for carefully reading the manuscript.

A. Feynman rules

We list below the complete Feynman rules for the model. All momenta are incoming and shown in the parentheses of the corresponding particles.

Propagators (momentum p):

$$A_\mu \quad A_\nu \quad = \frac{-i}{p^2}\left[g_{\mu\nu} - (1-\xi)\frac{p_\mu p_\nu}{p^2}\right]$$

$$Z_\mu \quad Z_\nu \quad = \frac{-i}{p^2 - m_Z^2}\left[g_{\mu\nu} - (1-\xi)\frac{p_\mu p_\nu}{p^2 - \xi m_Z^2}\right]$$

$$W_\mu^+ \quad W_\nu^- \quad = \frac{-i}{p^2 - m_W^2}\left[g_{\mu\nu} - (1-\xi)\frac{p_\mu p_\nu}{p^2 - \xi m_W^2}\right]$$

$$\sigma \quad \sigma \quad = \frac{i}{p^2 - m_\sigma^2}$$

$$\pi_0 \quad \pi_0 \quad = \frac{i}{p^2 - \xi m_Z^2}$$

$$\pi_+ \quad \pi_- \quad = \frac{i}{p^2 - \xi m_W^2}$$

$$c_A \quad \bar{c}_A \quad = \frac{i}{p^2}$$

$$c_Z \quad \bar{c}_Z \quad = \frac{i}{p^2 - \xi m_Z^2}$$

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$$= \frac{i}{p^2 - \xi m_W^2}$$

$G\phi\phi$ vertices:

$$\begin{aligned}
A_\mu \pi_+(p_+)\pi_-(p_-) &= \frac{ig}{\sqrt{2}}(p_+ - p_-)_\mu \exp(-ip_+ \wedge p_-) \\
Z_\mu \sigma(p_1)\sigma(p_2) &= \frac{g}{\sqrt{2}}(p_1 - p_2)_\mu \sin(p_1 \wedge p_2) \\
Z_\mu \pi_0(p_1)\pi_0(p_2) &= \frac{g}{\sqrt{2}}(p_1 - p_2)_\mu \sin(p_1 \wedge p_2) \\
Z_\mu \sigma(p)\pi_0(p_0) &= \frac{g}{\sqrt{2}}(p - p_0)_\mu \cos(p \wedge p_0) \\
W^\pm_\mu \sigma(p)\pi_\mp(p_\mp) &= \pm\frac{ig}{2}(p - p_\mp)_\mu \exp(\mp ip \wedge p_\mp) \\
W^\pm_\mu \pi_0(p_0)\pi_\mp(p_\mp) &= \frac{g}{2}(p_\mp - p_0)_\mu \exp(\mp ip_0 \wedge p_\mp)\,.
\end{aligned} \tag{A.1}$$

$GG\phi$ vertices:

$$\begin{aligned}
W^\pm_\mu(p_\pm)A_\nu(p)\pi_\mp &= \frac{ig^2 v}{2\sqrt{2}} g_{\mu\nu} \exp(\pm ip_\pm \wedge p) \\
W^\pm_\mu(p_\pm)Z_\nu(p)\pi_\mp &= \frac{ig^2 v}{2\sqrt{2}} g_{\mu\nu} \exp(\mp ip_\pm \wedge p) \\
W^+_\mu(p_+)W^-_\nu(p_-)\sigma &= \frac{ig^2 v}{2} g_{\mu\nu} \exp(ip_+ \wedge p_-) \\
Z_\mu(p_1)Z_\nu(p_2)\sigma &= ig^2 v g_{\mu\nu} \cos(p_1 \wedge p_2)\,.
\end{aligned} \tag{A.2}$$

$GG\phi\phi$ vertices:

$$\begin{aligned}
A_\mu(k_1)A_\nu(k_2)\pi_+(p_+)\pi_-(p_-) &= ig^2 g_{\mu\nu} \cos(k_1 \wedge k_2) \exp(-ip_+ \wedge p_-) \\
Z_\mu(k_1)Z_\nu(k_2)\sigma(p_1)\sigma(p_2) &= ig^2 g_{\mu\nu} \cos(k_1 \wedge k_2) \cos(p_1 \wedge p_2) \\
Z_\mu(k_1)Z_\nu(k_2)\pi_0(p_1)\pi_0(p_2) &= ig^2 g_{\mu\nu} \cos(k_1 \wedge k_2) \cos(p_1 \wedge p_2) \\
Z_\mu(k_1)Z_\nu(k_2)\sigma(p)\pi_0(p_0) &= -ig^2 g_{\mu\nu} \cos(k_1 \wedge k_2) \sin(p \wedge p_0) \\
W^+_\mu(k_+)W^-_\nu(k_-)\pi_+(p_+)\pi_-(p_-) &= \frac{ig^2}{2} g_{\mu\nu} \exp(-ik_+ \wedge k_-) \exp(-ip_+ \wedge p_-) \\
W^+_\mu(k_+)W^-_\nu(k_-)\sigma(p_1)\sigma(p_2) &= \frac{ig^2}{2} g_{\mu\nu} \exp(ik_+ \wedge k_-) \cos(p_1 \wedge p_2) \\
W^+_\mu(k_+)W^-_\nu(k_-)\pi_0(p_1)\pi_0(p_2) &= \frac{ig^2}{2} g_{\mu\nu} \exp(ik_+ \wedge k_-) \cos(p_1 \wedge p_2) \\
W^+_\mu(k_+)W^-_\nu(k_-)\sigma(p)\pi_0(p_0) &= -\frac{ig^2}{2} g_{\mu\nu} \exp(ik_+ \wedge k_-) \sin(p \wedge p_0) \\
W^\pm(k_\pm)A_\nu(k)\sigma(p)\pi_\mp(p_\mp) &= \frac{ig^2}{2\sqrt{2}} g_{\mu\nu} \exp(\pm ik_\pm \wedge k) \exp(\pm ip_\mp \wedge p) \\
W^\pm(k_\pm)Z_\nu(k)\sigma(p)\pi_\mp(p_\mp) &= \frac{ig^2}{2\sqrt{2}} g_{\mu\nu} \exp(\mp ik_\pm \wedge k) \exp(\pm ip_\mp \wedge p)
\end{aligned}$$

$$W^{\pm}(k_{\pm})A_{\nu}(k)\pi_0(p)\pi_{\mp}(p_{\mp}) = \mp\frac{g^2}{2\sqrt{2}}g_{\mu\nu}\exp(\pm ik_{\pm}\wedge k)\exp(\pm ip_{\mp}\wedge p)$$
$$W^{\pm}(k_{\pm})Z_{\nu}(k)\pi_0(p)\pi_{\mp}(p_{\mp}) = \mp\frac{g^2}{2\sqrt{2}}g_{\mu\nu}\exp(\mp ik_{\pm}\wedge k)\exp(\pm ip_{\mp}\wedge p)\,. \quad \text{(A.3)}$$

GGG vertices:

$$\begin{aligned}
A_{\alpha}(k_1)A_{\beta}(k_2)A_{\gamma}(k_3) &= -\sqrt{2}g\sin(k_1\wedge k_2)P_{\alpha\beta\gamma}(k_1,k_2,k_3)\\
Z_{\alpha}(k_1)Z_{\beta}(k_2)Z_{\gamma}(k_3) &= -\sqrt{2}g\sin(k_1\wedge k_2)P_{\alpha\beta\gamma}(k_1,k_2,k_3)\\
A_{\mu}(k)W^+_{\alpha}(k_+)W^-_{\beta}(k_-) &= -\frac{ig}{\sqrt{2}}\exp(-ik_+\wedge k_-)P_{\mu\alpha\beta}(k,k_+,k_-)\\
Z_{\mu}(k)W^+_{\alpha}(k_+)W^-_{\beta}(k_-) &= +\frac{ig}{\sqrt{2}}\exp(+ik_+\wedge k_-)P_{\mu\alpha\beta}(k,k_+,k_-)\,,
\end{aligned} \quad \text{(A.4)}$$

where

$$P_{\alpha\beta\gamma}(k_1,k_2,k_3) = (k_1-k_2)_{\gamma}g_{\alpha\beta} + (k_2-k_3)_{\alpha}g_{\beta\gamma} + (k_3-k_1)_{\beta}g_{\gamma\alpha}\,.$$

Some simple properties of it are useful:

$$\begin{aligned}
&P_{\alpha\beta\gamma}(k_1,k_2,k_3) = -P_{\alpha\gamma\beta}(k_1,k_3,k_2) = -P_{\beta\alpha\gamma}(k_2,k_1,k_3) = -P_{\gamma\beta\alpha}(k_3,k_2,k_1)\,,\\
&P_{\alpha\beta\gamma}(k_1,k_2,k_3) + P_{\gamma\alpha\beta}(k_1,k_2,k_3) = P_{\beta\alpha\gamma}(k_1,k_3,k_2)\,.
\end{aligned} \quad \text{(A.5)}$$

$GGGG$ vertices:

$$\begin{aligned}
&A_{\alpha}(k_1)A_{\beta}(k_2)A_{\mu}(k_3)A_{\nu}(k_4) =\\
&\quad = Z_{\alpha}(k_1)Z_{\beta}(k_2)Z_{\mu}(k_3)Z_{\nu}(k_4)\\
&\quad = -i2g^2\{(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})\sin(k_1\wedge k_2)\sin(k_3\wedge k_4) +\\
&\qquad + (g_{\mu\beta}g_{\nu\alpha} - g_{\mu\nu}g_{\alpha\beta})\sin(k_3\wedge k_1)\sin(k_2\wedge k_4) +\\
&\qquad + (g_{\mu\nu}g_{\alpha\beta} - g_{\mu\alpha}g_{\nu\beta})\sin(k_1\wedge k_4)\sin(k_2\wedge k_3)\} \qquad \text{(A.6)}\\
&W^-_{\alpha}(k_1)W^-_{\beta}(k_2)W^+_{\mu}(k_3)W^+_{\nu}(k_4) =\\
&\quad = ig^2(2g_{\mu\nu}g_{\alpha\beta} - g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})\cos(k_1\wedge k_3 + k_2\wedge k_4)\\
&W^+_{\alpha}(k_+)W^-_{\beta}(k_-)A_{\mu}(k_1)A_{\nu}(k_2) =\\
&= \frac{ig^2}{2}\{(-2g_{\mu\nu}g_{\alpha\beta} + g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha})\cos(k_1\wedge k_2) +\\
&\qquad + (g_{\mu\beta}g_{\nu\alpha} - g_{\mu\alpha}g_{\nu\beta})3i\sin(k_1\wedge k_2)\}\exp(-ik_+\wedge k_-) \qquad \text{(A.7)}\\
&W^+_{\alpha}(k_+)W^-_{\beta}(k_-)Z_{\mu}(k_1)Z_{\nu}(k_2) =\\
&\quad = \frac{ig^2}{2}\{(-2g_{\mu\nu}g_{\alpha\beta} + g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha})\cos(k_1\wedge k_2) +\\
&\qquad + (g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})3i\sin(k_1\wedge k_2)\}\exp(ik_+\wedge k_-) \qquad \text{(A.8)}\\
&W^+_{\alpha}(k_+)W^-_{\beta}(k_-)Z_{\mu}(k_1)A_{\nu}(k_2) =\\
&\quad = \frac{ig^2}{2}(2g_{\mu\nu}g_{\alpha\beta} - g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})\exp(ik_+\wedge k_2)\exp(ik_-\wedge k_1)\,. \qquad \text{(A.9)}
\end{aligned}$$

$\phi\phi\phi$ vertices:

$$
\begin{aligned}
\sigma\sigma(p_1)\sigma(p_2) &= -i6\lambda v\cos(p_1\wedge p_2)\\
\sigma\pi_0(p_1)\pi_0(p_2) &= -i2\lambda v\cos(p_1\wedge p_2)\\
\sigma\pi_+(p_+)\pi_-(p_-) &= -i2\lambda v\exp(ip_+\wedge p_-)\,.
\end{aligned}
\tag{A.10}
$$

$\phi\phi\phi\phi$ vertices:

$$
\begin{aligned}
&\sigma(p_1)\sigma(p_2)\sigma(p_3)\sigma(p_4) =\\
&\quad= \pi_0(p_1)\pi_0(p_2)\pi_0(p_3)\pi_0(p_4)\\
&\quad= -i2\lambda[\cos(p_1\wedge p_2)\cos(p_3\wedge p_4)+\cos(p_1\wedge p_3)\cos(p_2\wedge p_4)+\\
&\qquad\quad+\cos(p_1\wedge p_4)\cos(p_2\wedge p_3)]
\end{aligned}
\tag{A.11}
$$

$$\tag{A.12}$$

$$
\begin{aligned}
&\sigma(p_1)\sigma(p_2)\pi_0(k_1)\pi_0(k_2) =\\
&\quad= -i2\lambda\{2\cos(p_1\wedge p_2)\cos(k_1\wedge k_2)-\cos(p_1\wedge k_1+p_2\wedge k_2)\}\\
&\pi_+(p_1)\pi_+(p_2)\pi_-(k_1)\pi_-(k_2) =\\
&\quad= -i4\lambda\cos(p_1\wedge k_1+p_2\wedge k_2)
\end{aligned}
\tag{A.13}
$$

$$
\begin{aligned}
&\sigma(p_1)\sigma(p_2)\pi_+(k_+)\pi_-(k_-) =\\
&\quad= \pi_0(p_1)\pi_0(p_2)\pi_+(k_+)\pi_-(k_-)\\
&\quad= -i2\lambda\cos(p_1\wedge p_2)\exp(ik_+\wedge k_-)
\end{aligned}
\tag{A.14}
$$

$$
\begin{aligned}
&\sigma(p)\pi_0(k_0)\pi_+(k_+)\pi_-(k_-) =\\
&\quad= -i2\lambda\sin(p\wedge k_0)\exp(ik_+\wedge k_-)
\end{aligned}
\tag{A.15}
$$

$\phi c\bar{c}$ vertices:

$$
\begin{aligned}
\sigma c_Z(p)\bar{c}_Z(\bar{p}) &= -i\xi g^2 v/2\ \cos(\bar{p}\wedge p)\\
\sigma c_\pm(p)\bar{c}_\mp(\bar{p}) &= -i\xi g^2 v/4\ \exp(\mp i\bar{p}\wedge p)\\
\pi_0 c_Z(p)\bar{c}_Z(\bar{p}) &= -i\xi g^2 v/2\ \sin(\bar{p}\wedge p)\\
\pi_0 c_\pm(p)\bar{c}_\mp(\bar{p}) &= \pm\xi g^2 v/4\ \exp(\mp i\bar{p}\wedge p)\\
\pi_\pm c_\mp(p)\bar{c}_Z(\bar{p}) &= -i\xi g^2 v\sqrt{2}/4\ \exp(\mp i\bar{p}\wedge p)\\
\pi_\pm c_A(p)\bar{c}_\mp(\bar{p}) &= -i\xi g^2 v\sqrt{2}/4\ \exp(\mp i\bar{p}\wedge p)\,.
\end{aligned}
\tag{A.16}
$$

$Gc\bar{c}$ vertices:

$$
\begin{aligned}
A_\mu c_A(p)\bar{c}_A(\bar{p}) &= \sqrt{2}g\bar{p}_\mu\sin(\bar{p}\wedge p)\\
A_\mu c_\pm(p)\bar{c}_\mp(\bar{p}) &= \mp ig/\sqrt{2}\ \bar{p}_\mu\exp(\pm i\bar{p}\wedge p)\\
Z_\mu c_Z(p)\bar{c}_Z(\bar{p}) &= \sqrt{2}g\bar{p}_\mu\sin(\bar{p}\wedge p)\\
Z_\mu c_\pm(p)\bar{c}_\mp(\bar{p}) &= \pm ig/\sqrt{2}\ \bar{p}_\mu\exp(\mp i\bar{p}\wedge p)\\
W^\pm_\mu c_\mp(p)\bar{c}_A(\bar{p}) &= \mp ig/\sqrt{2}\ \bar{p}_\mu\exp(\pm i\bar{p}\wedge p)
\end{aligned}
$$

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$$
\begin{aligned}
W^{\pm}_{\mu} c_A(p)\bar{c}_{\mp}(\bar{p}) &= \pm ig/\sqrt{2}\ \bar{p}_{\mu} \exp(\mp i\bar{p}\wedge p)\\
W^{\pm}_{\mu} c_{\mp}(p)\bar{c}_{Z}(\bar{p}) &= \pm ig/\sqrt{2}\ \bar{p}_{\mu} \exp(\mp i\bar{p}\wedge p)\\
W^{\pm}_{\mu} c_Z(p)\bar{c}_{\mp}(\bar{p}) &= \mp ig/\sqrt{2}\ \bar{p}_{\mu} \exp(\pm i\bar{p}\wedge p)\,. \qquad \text{(A.17)}
\end{aligned}
$$

Counterterms for self-energies and mixings are listed below. Note the momentum p is the incoming momentum of the gauge boson in the $G\phi$ mixing.

$$
\begin{aligned}
&\sigma\ \text{- - - - - - - -}\times = i\lambda v^3\left[\frac{\delta\mu^2}{\mu^2} - \delta Z_\lambda - \frac{2\delta v}{v}\right]\\
&\sigma\ \text{- - - -}\times\text{- - - -}\ \sigma = ip^2\delta Z_\phi - im_\sigma^2\left[-\frac{\delta\mu^2}{2\mu^2} + \frac{3}{2}\delta Z_\lambda + \frac{3\delta v}{v}\right]\\
&\pi_0\ \text{- - - -}\times\text{- - - -}\ \pi_0 = ip^2\delta Z_\phi - im_\sigma^2\left[-\frac{\delta\mu^2}{2\mu^2} + \frac{1}{2}\delta Z_\lambda + \frac{\delta v}{v}\right]\\
&\pi_+\ \text{- - - -}\times\text{- - - -}\ \pi_- = ip^2\delta Z_\phi - im_\sigma^2\left[-\frac{\delta\mu^2}{2\mu^2} + \frac{1}{2}\delta Z_\lambda + \frac{\delta v}{v}\right]\\
&A_\mu \sim\sim\sim\times\sim\sim\sim A_\nu = i(p_\mu p_\nu - p^2 g_{\mu\nu})\delta Z_G\\
&Z_\mu \sim\sim\sim\times\sim\sim\sim Z_\nu = i(p_\mu p_\nu - p^2 g_{\mu\nu})\delta Z_G + ig_{\mu\nu}m_Z^2\left[2\delta Z_g + \frac{2\delta v}{v} + \delta Z_\phi\right]\\
&W^+_\mu \sim\sim\sim\times\sim\sim\sim W^-_\nu = i(p_\mu p_\nu - p^2 g_{\mu\nu})\delta Z_G + ig_{\mu\nu}m_W^2\left[2\delta Z_g + \frac{2\delta v}{v} + \delta Z_\phi\right]\\
&Z_\mu \sim\sim\sim\times\text{- - - -}\ \pi_0 = m_Z p_\mu\left[\delta Z_g + \frac{\delta v}{v} + \delta Z_\phi\right]\\
&W^\pm_\mu \sim\sim\sim\times\text{- - - -}\ \pi_\mp = \pm i m_W p_\mu\left[\delta Z_g + \frac{\delta v}{v} + \delta Z_\phi\right]\\
&c_A \rightarrow\times\leftarrow \bar{c}_A = ip^2\delta Z_c\\
&c_Z \rightarrow\times\leftarrow \bar{c}_Z = ip^2\delta Z_c - i\xi m_Z^2\left[\delta Z_g + \frac{\delta v}{v} + \delta Z_c\right]\\
&c_\pm \rightarrow\times\leftarrow \bar{c}_\mp = ip^2\delta Z_c - i\xi m_W^2\left[\delta Z_g + \frac{\delta v}{v} + \delta Z_c\right]
\end{aligned}
$$

The counterterms for vertices are obtained by attaching the following factors to the corresponding Feynman rules.

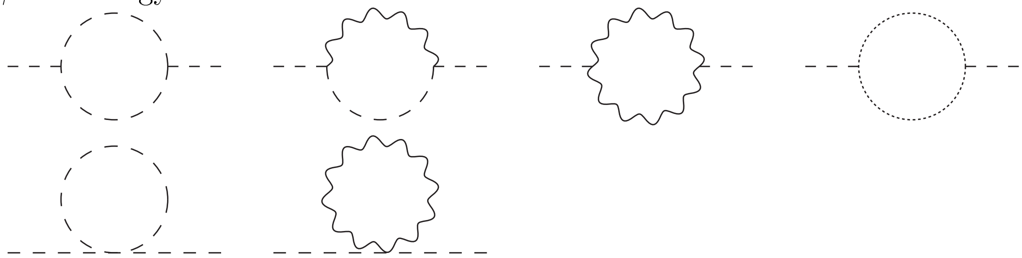
$$
\begin{aligned}
G\phi\phi &: (Z_\phi Z_g - 1) = \delta Z_\phi + \delta Z_g\,,\\
GG\phi &: [Z_\phi Z_g^2(1+\delta v/v) - 1] = \delta Z_\phi + 2\delta Z_g + \delta v/v\,,\\
GG\phi\phi &: (Z_\phi Z_g^2 - 1) = \delta Z_\phi + 2\delta Z_g\,,\\
GGG &: (Z_G Z_g - 1) = \delta Z_G + \delta Z_g\,,\\
GGGG &: (Z_G Z_g^2 - 1) = \delta Z_G + 2\delta Z_g\,,\\
\phi\phi\phi &: [Z_\lambda(1+\delta v/v) - 1] = \delta Z_\lambda + \delta v/v\,,\\
\phi\phi\phi\phi &: (Z_\lambda - 1) = \delta Z_\lambda\,,\\
\phi c\bar{c} &: (Z_c Z_g - 1) = \delta Z_c + \delta Z_g\,,
\end{aligned}
$$

$$Gc\bar{c} : (Z_c Z_g - 1) = \delta Z_c + \delta Z_g \,. \tag{A.18}$$

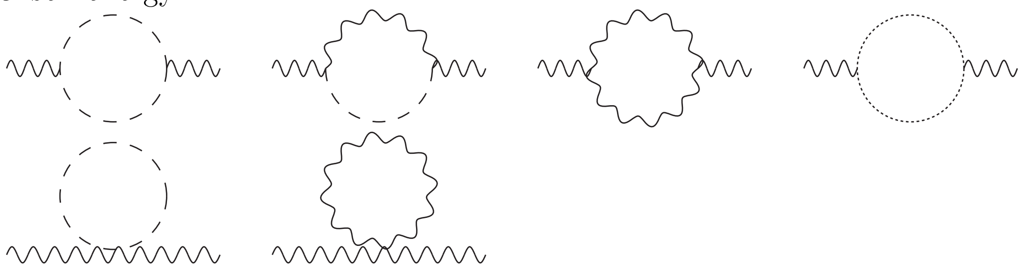
B. One loop diagrams for 1PI functions

We show below topologically different diagrams in which the wavy, dashed and dotted lines represent the gauge, scalar and ghost fields respectively. For a concrete vertex all possible assignments of fields must be included. Diagrams with an " f " are finite by power counting.

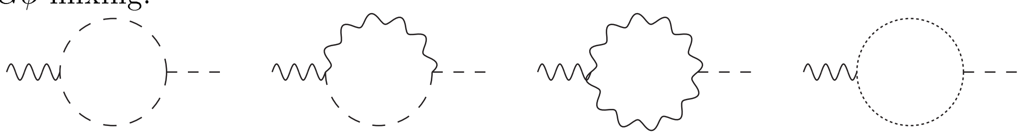
ϕ self-energy:



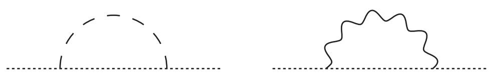
G self-energy:



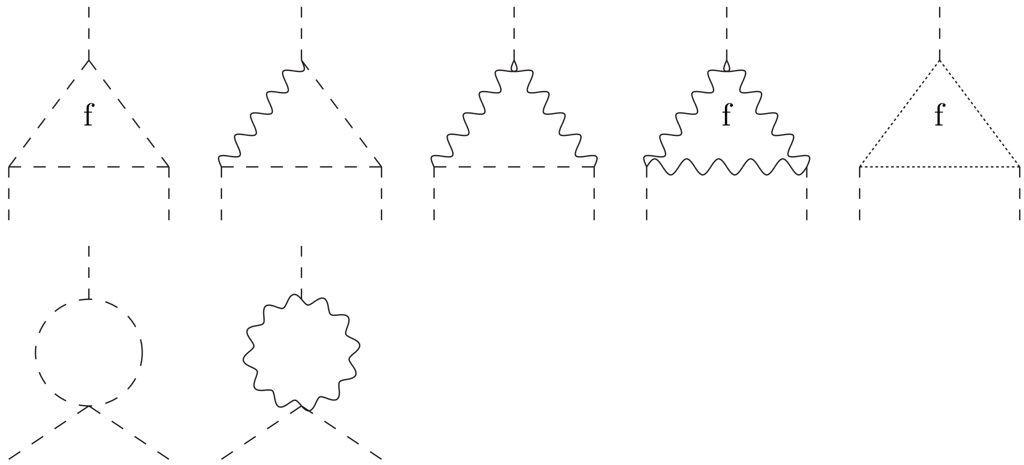
$G\phi$ mixing:



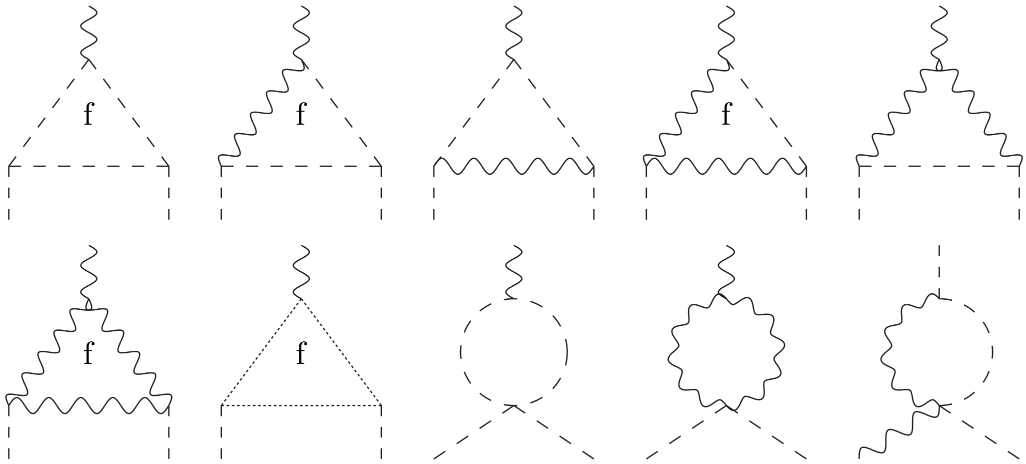
c self-energy:



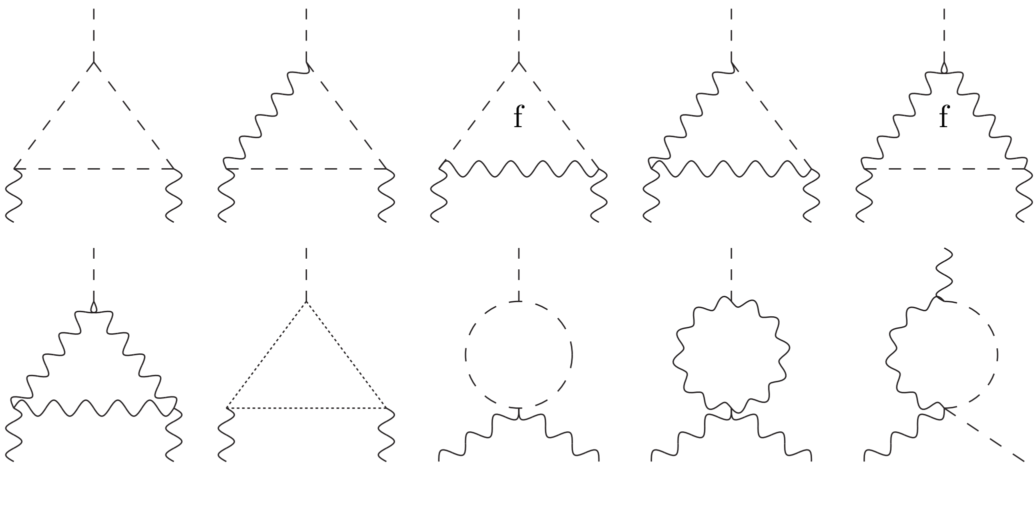
$\phi\phi\phi$ vertex:



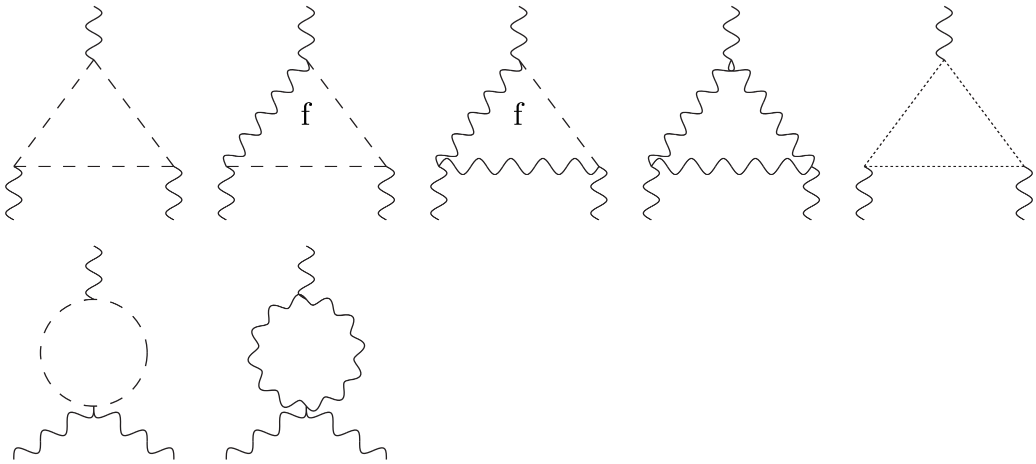
$G\phi\phi$ vertex:



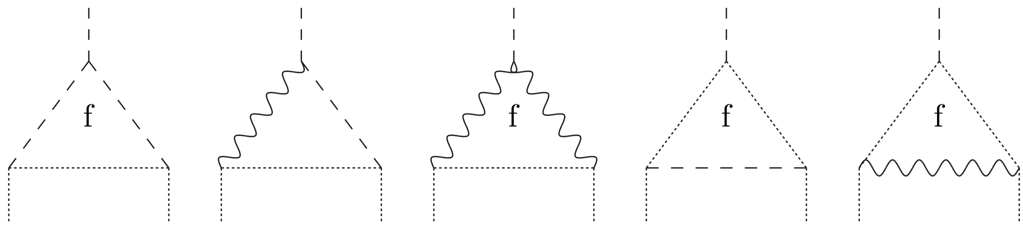
$GG\phi$ vertex:



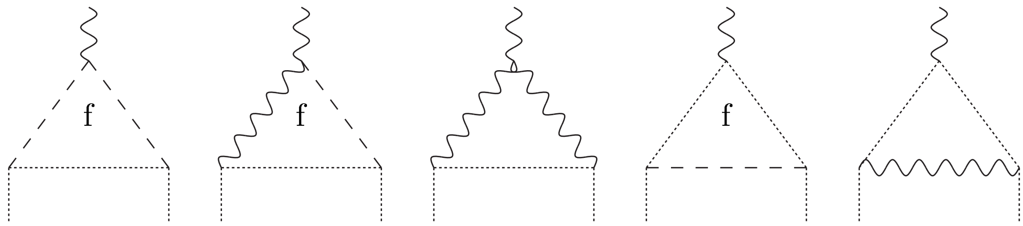
GGG vertex:



$\phi c\bar{c}$ vertex:



$Gc\bar{c}$ vertex:



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